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PANORAMIC VIEW OF CHAMBA, WESTERN HIMALAYAS.

A complete hydro-electric station has recently been installed by The General Electric Co., Ltd., at Chamba, in the Western Himalayas. Some idea of the difficulties of transport in this type of country may be gained from the accompanying photograph. Chamba, situated on a plateau at the foot of the mountains, is approached through passes in the mountains varying in height from 8,000 to 17,000 feet.

An article on this installation appears in the present issue.

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The G.E.C. in 1940.

THE first article each year in the G.E.C. Journal hitherto has reviewed "Electrical Progress and Development" during the preceding twelve months. Present conditions make it impossible to publish such a record of 1940, for energies have been concentrated upon the problems of war. A very large proportion of the output of G.E.C. factories has been required for the fighting services; demands have been met also from such essential civil departments as those concerned with A.R.P., Food, Health, Mines, Transport, and the like; moreover, the vast resources of the Company have enabled it to make a worthy contribution to Great Britain's export trade.

Activity has been focused not alone upon production, but also upon the advancement of those branches of science which underlie design and manufacture. The G.E.C. Research Laboratories and the technical staff at the various works throughout the country have been engaged continually in the development of new methods which will ensure that equipment in ships, aeroplanes, guns and munitions of every kind shall be the finest which skill can devise.

While it is possible only to generalise when writing of direct war effort, more specific reference may be made to export work. Soon after the outbreak of war, the directors of the Company laid down a policy of further intensification of its export effort, and this policy was endorsed subsequently by the Government. As a result, the volume of overseas trade has shown quite a satisfactory increase, and work is in hand for countries in every part of the world. An outstanding event of the year in New Zealand was the "New Zealand Centennial Exhibition." An important contribution to its success was the scheme of illumination for which use was made of 11,000 Osram lamps, 280 Osira electric discharge lamps, nearly a mile of fluorescent tubing, and large numbers of fittings, as well as considerable quantities of cable.

The country of Iraq is being developed rapidly, and the Company's Telephone Works is engaged in manufacturing for the city of Baghdad one of the largest public automatic telephone exchanges in the near East, which ultimately will provide for 10,400 subscribers. At this Works were manufactured also the radio receivers for the new broadcasting station in Johannesburg which receives and re-broadcasts programmes from Europe and the British Empire.

The overseas demand for generating plant, mercury arc rectifiers, transformers, switchgear and motors reveals great confidence in British built electrical machinery and equipment. As illustrating this demand, comprehensive electrical equipments are being installed for the operation of grain elevators in South America, motors are being built for various mining companies in South Africa and West Africa; air cooled pumpless steelclad rectifiers are in course of construction for towns in India, New Zealand and Western Australia; switchgear schemes are in hand for an industrial company in Turkey, for railways in India and South America, and also for a South African municipality. A direct current generator to be driven by a Fraser & Chalmers oil engine is destined for electricity supply in Macao, and turbo-alternators are required for installation in South Africa and India. High voltage transformers of considerable outputs will form part of the equipment of large transmission schemes in New Zealand, while several transformers have been built for service in the Federated Malay States.

Evidence is thus afforded of the ability of British industry to compete in world markets even when industrial production for war purposes is on an unprecedented scale.

The High Rupturing Capacity Cartridge Fuse.

WITH SPECIAL REFERENCE TO SHORT-CIRCUIT PERFORMANCE.

PART II.

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The first part of this paper, read before the Institution of Electrical Engineers in April, 1940, appeared in Vol. XI., No. 2 of the G.E.C. Journal (August, 1940).

In this second part conclusions are drawn as to the effect of circuit conditions on the duty imposed on the fuses; the choice of materials is considered, also the types of fuse for low- and medium-voltage circuits and for high-voltage circuits. The paper concludes with some notes on the application of these fuses.

(6) EFFECT OF CIRCUIT CONDITIONS ON PERFORMANCE.

Energy Considerations.

It is now possible, on the assumption that the breaking-capacity limit is reached by Method (*a*) of the preceding Section, to investigate the connection between breaking capacity and the circuit constants. If a failure of Type (*b*) supervenes to limit the breaking capacity, then the deductions set out in the following Sections, based on energy considerations, are not obviously valid; nevertheless, experience shows that even in this instance the breaking capacity is influenced in approximately the same way by the circuit conditions.

Circuit resistance being ignored for the moment, the fuse must obviously be able to absorb at least the inductive energy stored in the circuit. In practice, the arc energy exceeds the inductive energy in a ratio, to be called the "arc energy ratio," which varies from approximately unity up to perhaps 6 or 7. The ratio alters little with changes in inductance, but depends on resistance and, more particularly, on the circuit voltage. It is shown in Appendix 4 that the lower the arc voltage in relation to the circuit voltage, the higher is the ratio, or, in other words, the greater is the "follow-through" energy which the fuse must absorb. The curves of fig. 17 give the results of tests in a D.C. circuit of variable voltage but fixed inductance and resistance, of the 60-amp. fuse for which test oscillograms are given in figs. 7 and 18. The rapid increase in arc energy ratio as the test voltage approaches the fuse arc voltage should be noted. The curves also illustrate the way in which peak current and inductive energy vary with rate of rise of current in a quick-acting fuse.

Relation between Inductive Energy and Inductance.

The peak value of the energy stored in the inductance of the circuit is given by

$$W = \tfrac{1}{2}LI_f^2 \quad . \quad . \quad . \quad . \quad (1)$$

(*a*) If the fuse is quick-acting, then (see Appendix 2)

$$I_f \propto E^{\frac{1}{2}}L^{-\frac{1}{2}}$$

so that, from (1),

$$W \propto L(E^{\frac{1}{2}}L^{-\frac{1}{2}})^2$$
$$\propto E^{\frac{1}{2}}L^{\frac{1}{2}}$$

For direct current, the energy is independent of prospective current. For alternating current, provided that the short-circuit power factor is low,

$$I_s \propto EL^{-1}$$

and

$$W \propto EI_s^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (2)$$

(*b*) If the fuse is slow-acting, allowing the full prospective current to develop, then, from (1),

For direct current,

$$W \propto E^2L$$

For alternating current, provided that the short-circuit power factor is low,

$$I_f \propto EL^{-1}$$

so that

$$W \propto L(EL^{-1})^2$$
$$\propto E^2L^{-1} \quad . \quad . \quad . \quad (3)$$

i.e., for a constant voltage

$$W \propto I_s$$

TABLE 1.

Circuit quantities	Effect on severity of increase of circuit quantities	
	Quick-acting fuse	Slow-acting fuse
Inductance, assuming changes in arc energy ratio to be negligible	Severity increases as the cube root of inductance. For A.C., equation (2) shows that it varies inversely as the cube root of prospective current	For D.C., severity increases directly with inductance. For A.C., equation (3) shows that it varies inversely as inductance and so directly with prospective current
Resistance	While peak current is unaffected, severity decreases through decrease in arc energy ratio	Severity decreases both by decrease in peak current (this being small with A.C.) and by decrease in arc energy ratio
Voltage	The inductive energy increases as the $\frac{3}{2}$ power of voltage, and the severity at a greater rate through the simultaneous increase in arc-energy ratio. This is well shown in Fig. 17	The inductive energy increases as the square of voltage, and the severity at an even greater rate

Relation between Severity, based on Arc Energy, and the Circuit Constants.

It is now possible, by considering variations in inductive energy and in the arc energy ratio, to assess the effect on severity of changing the circuit constants one at a time. Quick-action and slow-action are considered separately in Table 1, it being understood that any particular fuse may act in one manner or the other, or in an intermediate manner, depending on the circuit constants.

It is very noteworthy that for a quick-acting fuse, provided that circuit resistance is low, decrease in A.C. prospective current results in an increase of arc energy, the maximum value of which is reached when further reduction in prospective current causes the current-limiting effect to be lost. Inversion of severity is, however, not necessarily of much importance in service, since a low value of prospective current at a particular point on a network is, in general, partly or chiefly the result of high resistance, which prevents inversion from taking place. Inversion is also likely to occur with direct current if a reduction of prospective current is accompanied by an increase of inductance. An example is given in Table 2, which relates to two tests on a certain 60-amp. fuse at the same voltage, but at different prospective currents. The first test, illustrated in fig. 7, is a direct short-circuit on a large motor-generator set, while fig. 1 represents a direct short-circuit on a much smaller set.

In both tests the short-circuit was cleared in a satisfactory way, but in the second test the breaking capacity limit was more nearly approached, since the arc energy was higher.

It should be noted that the inversion of severity described above is entirely distinct from that mentioned under "Operation on Moderate Overloads" (Section 5), and in general occurs at much higher prospective-current values.

Phase Position of A.C. Short-Circuit.

The shaded areas in fig. 9 give the value of ampere-seconds absorbed during the arcing period, and, since the arc voltage is assumed constant, they also give a measure of arc energy. Table 3 summarizes the results shown by the curves.

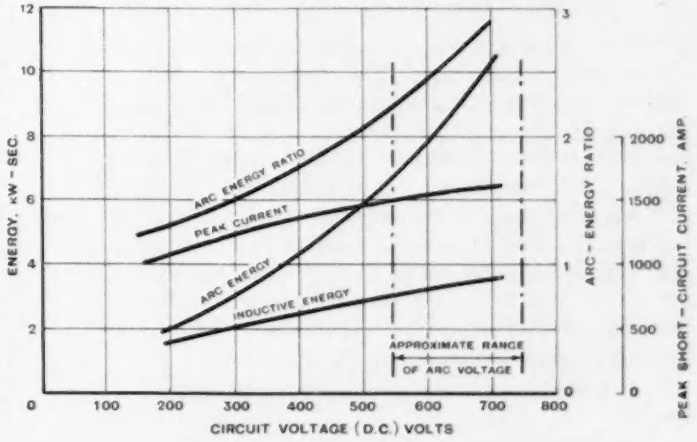

Fig. 17.—Relation between circuit voltage and arc energy. 60-amp. fuse subjected to short-circuit test in a circuit of the following constants :—

Resistance 0.124 ohm.
Inductance 2.73 millihenrys.

Thus the *severest condition arises when arcing commences a little before the peak of the supply voltage.* This conclusion is not invalidated by the two principal complications occurring in practice, namely the presence of circuit resistance and the rising voltage/current characteristic of the arc.

Comparison of A.C. Operation and D.C. Operation.

When the arcing time is short compared with the time of one half-cycle, A.C. operation can be regarded as the equivalent of D.C. operation in a circuit of voltage equal to the instantaneous A.C. circuit voltage during arcing. Thus A.C. operation may be very easy or may be as difficult as D.C. operation at a voltage equal to the peak value of the A.C. voltage. Variations of resistance and inductance have, as is shown earlier, less effect than proportional variations of voltage. If, however, these two quantities in an A.C. circuit are taken as equal to those in a D.C. circuit, and, further, if the R.M.S. A.C. voltage is equal to the D.C. voltage, then the arc energy for the A.C. case will exceed that for the D.C. in the ratio

TABLE 2.

	Fig. 7	Fig. 18
Rating of set, kW	2 × 1,270	2 × 125
D.C. test voltage	660	660
Circuit resistance, ohms ..	0.0175	0.124
Circuit inductance, millihenrys	0.22	2.73
Time-constant, sec.	0.0125	0.022
Peak current, amp.	3,850	1,580
Inductive energy, kW-sec. ..	1.63	3.41
Arc energy, kW-sec.	7.65	9.17
Arc energy ratio	4.69	2.69

$$\left(\frac{\text{Peak A.C. voltage}}{\text{R.M.S. A.C. voltage}}\right)^{\frac{n}{2}} = (\sqrt{2})^{\frac{n}{2}}$$

where n is dependent on the arc energy ratio and is always greater than unity. The comparison is more difficult for longer arcing periods, but it may be stated generally that, for any particular fuse, A.C. operation is more difficult than D.C. at the same voltage, and this conclusion is fully borne out in practice. Further, if short-circuit operation on a 3-phase A.C. system and on a 3-wire D.C. system, operating at the same voltage to earth, are compared, the former will be seen to present much the more stringent conditions since, in addition to the inherently greater difficulty of A.C. operation, the voltage-recovery conditions, as shown in Section (2), are severer.

It is impossible to lay down any definite rule for the relation between breaking capacities on alternating current and on direct current which will take into full account the several variables involved. The author therefore considers it impracticable to formulate proving tests on alternating current only or on direct current only for fuses to be used on both types of system.

Comparison of Duty Categories of British Standard 88—1939.

It is now possible to consider the effect on performance of the changes of circuit conditions expressed in the Specification in terms of current (D.C. or A.C.) at certain specified values of time-constant and power factor.

(a) Direct Current.

The last two columns in Table 4 are calculated from the specified values of current and time-constant. In proceeding from Category 1 to Category 4, the decrease in resistance tends towards the desired result of increasing the severity of the test, but the decrease in inductance has the contrary effect, so that whether in fact the conditions become progressively more severe with increase of prospective current depends on which of the two factors predominates. A quick-acting fuse is, as has been shown previously, likely to be little affected by changes of resistance, so that the decreasing inductance accompanying increased prospective current may result in tests at the higher categories stressing the fuse less than at the lower. Inversion cannot occur with a slow-acting fuse, since the decrease in inductance in ascending the scale of categories is more than outweighed by the increase in peak current.

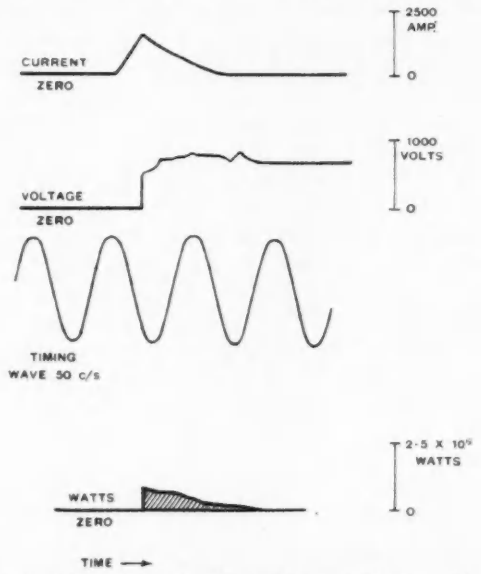

Fig. 18.—D.C. short-circuit on the same 60-amp. fuse as in Fig. 7, but with lower value of prospective current.
Shaded area indicates energy absorbed by fuse.

(b) Alternating Current.

In order to give a direct comparison with the categories of D.C duty, the 250-volt series is enumerated in Table 5.

Here the resistance values are much lower than in the corresponding D.C. categories, so that inductance has a dominating effect. Thus an inversion of severity between any two categories can be expected

if action is "quick" at the lower category as well as at the higher. Since in general the smaller the current rating of a fuse, the lower the short-circuit current at which current-restriction occurs, it follows that while Category 4 represents the severest test on a fuse of high current rating, a smaller fuse may find Category 3 or even Category 2 the most difficult.

TABLE 3.

Fig.	Phase angle of initiation of short-circuit, relative to zero of circuit e.m.f.	Circuit e.m.f. at initiation of short-circuit	D.C. component of s.c. current	Peak current	Severity of short-circuit
9(*a*)	0°	Zero	Maximum	Minimum	Intermediate
9(*b*)	45°	Rising	Intermediate	Intermediate	Maximum
9(*c*)	90°	Maximum	Zero	Maximum	Intermediate
9(*d*)	135°	Falling	Intermediate	Intermediate	Minimum

(7) HEATING AND FUSING CHARACTERISTICS.

The temperature rise of any particular fuse must always be a matter of test, since the complicated nature of the enclosure surrounding the element, which includes several different materials, together with the end effect makes calculation nearly impossible. It is, however, possible to determine the temperature rise of the element itself, since this depends only on the material and the fusing-factor. Experimental curves giving such figures are shown in fig. 19. The low temperature-rises at fusing factors of the order of 1·5 to 2 are noteworthy and help to explain how a high-melting-point metal can be used at such fusing factors without the fuse as a whole running excessively hot. The effect in question is a result largely of the increase of resistance with temperature.

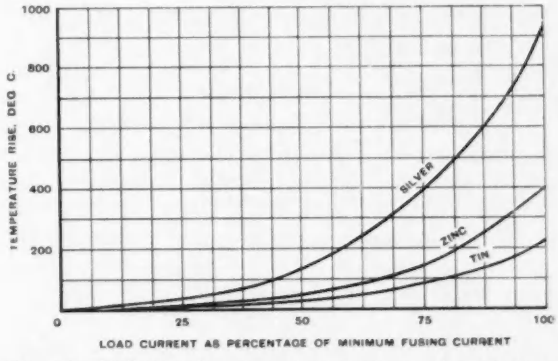

Fig. 19.—Relation between temperature rise of fuse-element and load current.

Ambient temperature = 20 degs. C.

But because the cartridge fuse has, in general, a small section of element it is necessary to take special steps to minimize the power loss. In the case of low- and medium-voltage fuses, the element is short compared with that of a semi-enclosed fuse and this helps to offset the reduced section. The use of a necked element is of great assistance; although the section of the neck is somewhat less than that of a straight-through element of the same rating, yet the rest of the element has a considerably greater section, the net result being a valuable diminution of resistance.

In general, the use of a high-melting-point element results in a rather higher full-load loss than that with low-melting-point material, but the difference is relatively small, on account of the large variation of resistance between the running temperature and the melting point; this variation has already been shown to be also of very great service in reducing cut-off currents on short-circuit.

The fusing-current is given by an expression of the form kd^n. The value of the coefficient n was given by Preece[1] as 1·5 and by Schwartz and James[2] as between 1·19 and 1·46. For those cartridge fuses where the end effect is slight, the author has found the older figure of 1·5 to be reasonably accurate, and this is probably because the presence of the powder filling results in the effect of conduction predominating over that of convection.

TABLE 4.

Category	Prospective current	Time-constant	Resistance	Inductance
	amp.	sec.	ohm	mH
DC1	1,000	0.003	0.25	0.75
DC2	4,000	0.004	0.0625	0.25
DC3	16,500	0.006	0.0152	0.091
DC4	33,000	0.0075	0.0076	0.057

Voltage = 250 volts.

The factors determining the time-lag of cartridge fuses at various overload values have been investigated in detail by Wilson.[12]

(8) CHOICE OF MATERIALS.

Metals for Fuse Elements.

Composite fuse elements being ignored for the present, the chief requirements for a metal for a high-rupturing-capacity fuse are:—

(i) A high value of Preece's coefficient as stated in Section (3). (This excludes the use of low-melting-point metals such as tin.)

(ii) Freedom from deterioration, as by oxidation.

(iii) Low specific heat, to assist in current restriction.

(iv) Freedom from formation of an oxide "skin" on fusing, liable to give trouble at moderate overloads. A skin is, however, less likely to be formed in an enclosed fuse on account of the small amount of air available.

(v) High temperature coefficient of resistance.

TABLE 5.

Category	Prospective current	Power factor	Resistance	Inductance
	amp.		ohm	mH (50 c./s.)
250AC1	1,000	0.6	0.15	0.637
250AC2	4,000	0.4	0.025	0.1825
250AC3	16,500	0.3	0.00456	0.0460
250AC4	33,000	0.3	0.00228	0.0230

Voltage = 250 volts.

The metals principally used are silver, copper, aluminium and zinc. Experimental curves connecting peak current on short-circuit and minimum fusing-current, for various metals, are given in fig. 20. They apply to a circuit of one specific value of rate of current rise, though a change of rate does not alter the relative positions of the curves. The wires tested were in air, but the general shape of the curves is not sensibly changed by the powder filling of a fuse.

Silver, apart from its high cost, is on the whole the best metal for fuse elements. It is very free from oxidation; and although its value of Preece's coefficient is rather lower than that of copper, necessitating a somewhat greater section for a given current, this factor is more than outweighed as regards breaking capacity by the lower specific heat of silver, resulting in more pronounced current restriction. Its lower Preece's coefficient and its lower resistivity result in an ohmic loss which is less than for copper, and does not greatly exceed that for tin.

Aluminium, which has sometimes been used, has the disadvantage of forming a very marked skin. In certain bimetallic fuses which will be considered in the next Section, silver or copper are combined with low-melting-point alloy.

Fillers.

There are two general types :—

(i) The inert filler which in being fused by the arc gives rise to only limited chemical action. The most usual material of this sort is silica (SiO_2) in one or other of its forms; the chemical action involved is the formation of small quantities of the silicate of the element metal.

(ii) The filler which gives off gas when heated by the arc, the action being endothermic. Pressure-relief vents are generally used, and need careful design to prevent emission of flame and a too-free release. The commonest example of this class of filler is calcium carbonate ($CaCO_3$) in the form of powdered or granulated marble, the gas evolved being CO_2, which is believed to assist in arc extinction.

Bodies.

High mechanical strength is usually considered necessary, and for this reason bodies are commonly made of ceramic material of a grade having special resistance to heat and internal pressure. On the other hand, at any rate one design of high-performance fuse successfully employs a comparatively light tube of fibre, in conjunction with pressure relief; in some ratings the gases are discharged into a separate chamber, formed by the annular space between the main fuse tube and a surrounding outer tube.

(9) AVAILABLE TYPES OF FUSE.

In considering the principles of operation, it has not been necessary to make any distinction between

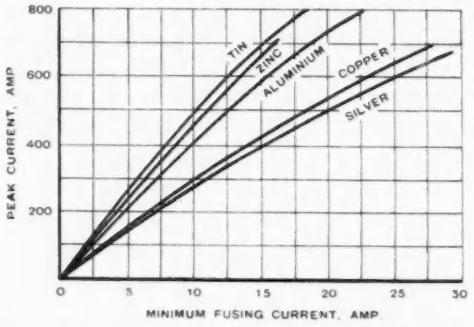

Fig. 20.—Relation between I_f and I_c for different sizes and material of wire. Fuse wire in asbestos tube in ordinary semi-enclosed porcelain holder. Rate of rise of current = 550 amp. per millisec.

low- and high-voltage fuses. Even the details of construction do not vary greatly between fuses for low and medium voltages on the one hand and for high voltages on the other; the principal differences arise from the necessity of using a longer element in the latter case.

Fuses for Low and Medium Voltages.

These can be divided into (*a*) those using elements of one metal only, and (*b*) those with composite elements.

(*a*) Where there is one metal only, silver is almost always chosen for its advantages already enumerated. Except for ratings below about 10–20 amp., sub-division of the elements is commonly employed. The paralleled elements are sometimes used in a single bored body, while other designs

employ a "honeycomb" type of body, each bore carrying only one or two elements. Occasionally the silver elements are paralleled by one or more fine wires of resistance alloy, which have been regarded as

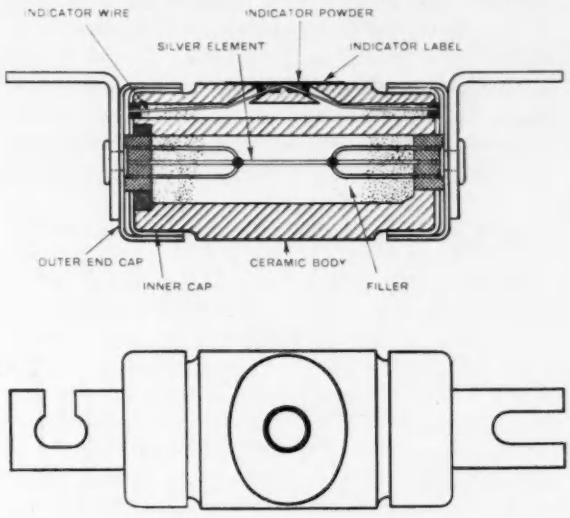

Fig. 21.—Typical silver element fuse-link for low or medium voltage.

assisting in short-circuit interruption. Most designs used necked elements to assist in cool running and to reduce voltage surges. Heat insulation may be provided at the neck or necks to reduce the total loss in the fuse. A typical design of fuse-link with silver elements is shown in fig. 21.

(*b*) The object of the composite-element construction is to afford longer time-lags on overload than are associated with simple silver elements, and to facilitate cool running and consequently a low fusing factor. In one form, a copper or composite copper/silver element is provided with two necks and with a complete gap bridged by a mass of a low-melting-point alloy containing tin and of large cross-section. Short-circuits are interrupted by fusion of the neck or necks, the alloy being left intact. Overload operation results from the alloy being melted by the heat generated at the necks, and the time-lag is largely dependent on the time necessary for the heat transfer. A typical design is shown in fig. 22. In a type introduced[25] more recently the element is of silver carrying a blob of solder. In overload operation, the silver reaches a temperature at which amalgamation between the silver and the solder takes place, and the resulting increase in resistance causes fusion. Short-circuits are cleared by the silver fusing in the normal manner.

Where indication of operation is required, a fine wire in parallel with the main elements is provided. Fusion of this wire either fires a small quantity of a mild explosive to give indication by charring a paper label, or releases a small indicating spring.

Fuses for High Voltage.

In some designs the arrangement is very similar to that used at lower voltages, but with an appropriately increased length of body. In others, particularly for 11 kV and upwards, the required length of element is obtained by winding in the form of a helix on a ceramic core of star-shaped cross-section running along the axis of the fuse tube, the space between the two members containing the filler. The helical arrangement would be ineffective with a fluid filler, but in a powder of suitable dielectric qualities the current continues to follow the helical path after arcing has commenced. Sometimes the element is of uniform section but is provided with heat insulation near the middle of its length to initiate fusion there under overload conditions. In other designs, while no definite neck is provided, there is a slight variation of section designed to reduce voltage surges produced by simultaneous fusion of the whole length of the element. In still other designs, reduction of surges is obtained by providing high-resistance parallel elements which further greatly increase their resistance before fusing, tungsten with its high melting-point of over 3,000 degs. C. being chosen

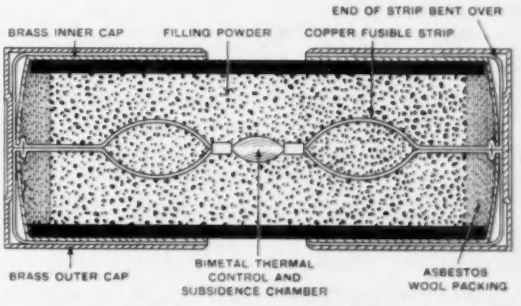

Fig. 22.—Typical bimetallic fuse-link for low or medium voltage.

for the purpose. A high-voltage fuse with helical element is shown in fig. 23.

In general, the necessity for special steps to reduce heating of the fuse makes itself felt less than at the lower voltages, because currents are lower and cooling is better, while higher fusing factors are usually preferred. On the other hand, possible trouble from corona, particularly with fine wires, has to be borne in mind; the use of a number of adjacent parallel wires or of a helical formation is of assistance by reducing the voltage gradient at the surface of the wire.

When indicators are fitted, it is necessary that they shall be visible at a distance if the fuse is to be used for outdoor work. Fusion of the indicator wire usually either releases a spring operating an indicating plunger or fires an explosive to expel the plunger. The movement can be used in some designs for remote indication, or to operate a switch which

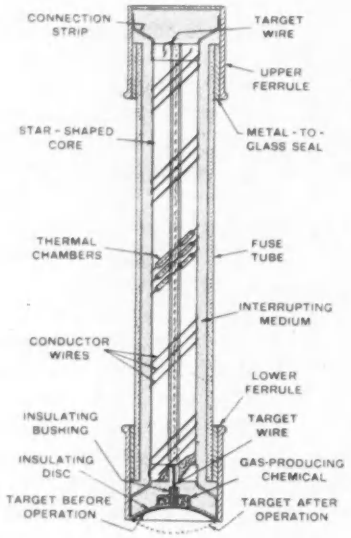

Fig. 23.—Typical helical-element high-voltage fuse-link.

prevents single-phasing by opening all phases upon the occurrence of a fault in one.

(10) APPLICATIONS.

General.

The principal uses of low- and medium-voltage H.R.C. cartridge fuses may be summarized as :—

(i) The sole line of protection on distribution boards in substations, the fuses being used in both main outgoing and auxiliary circuits.

(ii) In distribution networks. With cable systems the fuses are placed in feeder pillars or link disconnecting boxes, while for overhead work, special outdoor fittings are available for pole operation.

(iii) Service fuses in domestic and similar circuits, their function being to back up sub-circuit fuses.

(iv) In general industrial distribution, both in main factory feeders and for backing-up motor starters ; in the latter case the fuses can either be grouped in distribution boards or incorporated in the starters themselves. Protection of motor and other auxiliary circuits in power stations forms a similar field of application.

(v) For the backing-up of circuit-breakers of inadequate breaking capacity.

(vi) For certain special applications, such as (*a*) Mercury-arc-rectifier anode circuits. (*b*) Traction work, for the protection of auxiliary circuits on rolling stock. (*c*) The protection of power-factor-correction condensers, special designs for resisting switching-current peaks being available.

The applications of high-voltage cartridge fuses, though not so far very numerous in this country, are on the same general lines as for low- and medium-voltage fuses. For indoor work, they are usually placed in cubicles ; while for outdoor use, such as the protection of transformers for rural distribution, convenient pole-mounting arrangements are available.

Choice of Low- and Medium-Voltage Fuses on the Basis of Breaking Capacity.

While it can be broadly suggested[14] that B.S. Duty Categories 1 and 2 correspond to domestic service and Categories 3 and 4 to industrial service, yet the short-circuits encountered in practice vary so widely that it is not possible to make any definite suggestions as to the type of fuse required, as expressed by category of duty, for different types of service. For example, contrary to the suggestion made above, short-circuit values above 16,500 amp. are quite common in large cities at the supply point in residential buildings, resulting in Category 4 fuses being required.

Use of Fuses for Backing-up Motor Starters, Circuit-Breakers and other Fuses.

The usual requirement is that the H.R.C. fuses shall clear heavy short-circuits, leaving the other protective apparatus to deal with normal overloads. In meeting this requirement, two main questions are involved, namely discrimination and current-restriction ; both are considered below.

Discrimination.

(*a*) *Between one fuse and another.*—For satisfactory discrimination between a sub-circuit fuse and a main fuse, the requirement is obviously that the fusing-current/time curve for the former shall lie below the corresponding curve for the latter. If the fuses are of the same type, this demand is easily met by choosing fuses with an appropriate difference in their current ratings. Thus in factory distribution the H.R.C. fuses on a sub-distribution board are in series with those on the main distribution board and satisfactory discrimination can be obtained by ensuring that the ratings of the fuses decrease from the feeding point.

An important case is the protection of step-down transformers for, say, rural distribution work, where the fuses on the h.v. side are required to operate only with internal faults in the transformer, any faults or overloads on the secondary network being handled by the l.v. fuses. The correct fuse to use on the h.v. side can be determined by superimposing the curves

for h.v. and l.v. fuses, the current scales being chosen so as to make them directly comparable. A typical case is shown in fig. 24. While the curve for the h.v. fuse must lie above that for the fuse (or breaker) on the secondary side, it should preferably be below that for any fuse or circuit-breaker nearer to the supply. The fuses protecting the transformer must also be able to withstand the magnetizing-current peaks when switching on; this requirement may sometimes fix the rating of fuse at a higher value than that required simply for discrimination with the l.v. fuses.

Considerations of fusing time give a satisfactory indication of discrimination under conditions of overload and moderate short-circuit, where the fuse arcing time is short compared with the fusing time. At higher currents it becomes impossible by super-

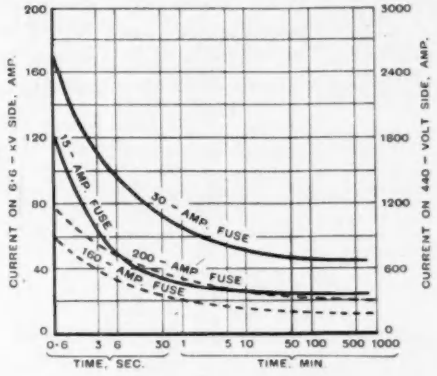

Fig. 24.—Discrimination between fuses in primary and secondary circuits of distribution transformer.
——— Curves for h.v. fuses.
– – – Curves for l.v. fuses.
Diagram shows that for 160-amp. fuse on l.v. side, 15-amp. fuse is required on h.v. side, and that for 200-amp. fuse on l.v. side, 30-amp. fuse is required on h.v. side.

imposing fusing-current curves to determine whether satisfactory discrimination can be obtained, since the arcing time has an uncertain effect. Most current/time curves for fuses give only the fusing time; it is difficult to include the arcing time, which depends on many different factors.

For discrimination between a sub-circuit fuse and a main fuse to be obtained at high currents, the current must reach its peak value and be reduced to zero by the sub-circuit fuse before the melting point of the element(s) in the main fuse is attained. Satisfactory discrimination usually results if the pre-arcing time of the main fuse for the short-circuit in question is equal to the total operating time of the sub-circuit fuse; this allows a margin, since initiation of arcing in the sub-circuit fuse reduces the heating in the main fuse. Discrimination therefore becomes less certain as the prospective current is increased, since the pre-arcing time is decreased while the arcing time usually remains about the same, and it thus becomes necessary to increase the rating of the main fuse.

Cartridge-type service fuses are commonly employed in domestic installations and are required to operate only on failure of a semi-enclosed sub-circuit fuse or in the somewhat rare event of a short-circuit on the supply side of the sub-circuit fuses. In such an installation, discrimination difficulties may arise on account of the large cross-section of element in a semi-enclosed fuse, in proportion to rating, and the somewhat indeterminate arcing time. The result is that the service fuses have usually to be of much higher rating than the sub-circuit fuses. The results of investigations by several manufacturers have recently been embodied in a recommendation of the Institution of Electrical Engineers[24] that where service fuses are of the quick acting (*i.e.*, H.R.C. cartridge) type the installation should be so sub-divided that the current rating of the largest consumer's fuse is not greater than one-third of the current rating of the service fuse. In the author's opinion, a figure of one-quarter would on the whole be preferable. It may, however, be pointed out that for this type of service, cartridge fuses are available in which the pre-arcing time is longer than usual, *i.e.*, the current-restricting effect is deliberately made less pronounced. If, as is technically preferable, cartridge fuses are used throughout, then the ratio of $\frac{1}{3}$ or $\frac{1}{4}$ can usually be increased to $\frac{1}{2}$ or even 1/1·5.

(*b*) *Between fuse and starter or circuit-breaker.*—It is usually required that the fuse shall operate above a certain current, and the starter or breaker below this current; the current/time curves of the two pieces of apparatus are therefore required to intersect at this current,[14] this being easy to achieve since the starter or breaker always has a well-defined minimum operating time (including that of the relays) of at least several cycles.

Starters cannot be expected[8] to be capable of dealing with currents much in excess of the motor standstill current, and this requirement is generally satisfied, in the case of direct starting of squirrel-cage motors, if the 12-sec. fusing current of the fuse exceeds the motor standstill current by 10 per cent or somewhat more. Where star-delta or auto-transformer starting is used, it is undesirable to reduce the fuse rating in proportion to the reduction in starting current; considerable peaks may be encountered in switching from the intermediate to the full voltage position, and the author considers that the fuse rating should not be reduced below 70 per cent of that for direct starting.

Where the short-circuit capacity of a system has outgrown existing circuit-breakers, the addition of H.R.C. fuses can be made to ensure that while the breakers are left to deal with ordinary overloads in the normal way, they do not have to open under extreme short-circuit conditions; similarly the cost of new installations can often be substantially

reduced through allowing the use of breakers of lower breaking-capacity. In either case, correct discrimination is essential and is ensured in the usual way by consideration of the current/time curves; the intersection point is higher in proportion to normal load than with starters on account of the higher breaking-capacity of circuit-breakers.

(*c*) *In mercury-arc-rectifier anode circuits.*—Cartridge fuses are frequently used for high-speed protection of rectifiers. The choice of rating for such service requires considerable care, since the best possible compromise is required between two requirements, namely that the fuses shall remain intact when short-circuits occur on the D.C. system, which should be dealt with by the feeder breakers; but that they shall operate as quickly as possible if a backfire should occur. Experience shows, however, that if the D.C. breakers are of the polarized high-speed type, they usually operate sufficiently quickly for the fuses to withstand the effect of external short-circuits. Adequate protection against backfires can then be obtained by selecting a fuse simply on the basis of the same percentage-overload and time figures as are specified for the rectifier itself.

Influence of Current Restriction on Back-up Protection of Starters and Circuit-Breakers.

(*a*) *Starters.*—It is not entirely sufficient that the fuses shall relieve the starter of attempting to break short-circuits; its contacts and magnetic or thermal trips should also be protected as far as possible against the destructive mechanical and thermal effects of the current. The protection of this sort resulting from the current-restricting effect of the fuses is more satisfactory the lower their rating. It is therefore desirable to sub-divide the circuits so that one set of fuses feeds only one, or at any rate only a very few, motors. The curves in fig. 3 can be used to determine the likely peak currents. For example, short-circuit capacities as high as 25 MVA, corresponding to 84 kA peak asymmetrical current, may be encountered in 440-volt 3-phase systems;

LIST OF SYMBOLS.

I_s = prospective current (D.C.) or R.M.S. symmetrical prospective current (A.C.).
I_{fp} = instantaneous value of current during pre-arcing period.
I_{fa} = instantaneous value of current during arcing period.
I_{fs} = instantaneous value of prospective current.
I_f = cut-off current.
I_c = minimum fusing current.
$i = I_{fp}/A$ = instantaneous value of current density during pre-arcing period.
i_0 = prospective current density (D.C.) or symmetrical peak value of prospective current density (A.C.)
B = rate of rise of current in quick-acting fuse during pre-arcing period.
θ = temperature of fuse element, ambient temperature as datum.
θ' = apparent temperature of fuse element, ambient temperature as datum.
θ_m = melting temperature, ambient temperature as datum.
θ_v = vaporization temperature, ambient temperature as datum.
$\Delta\theta$ = temperature rise above melting point.
c = specific heat, in joules per cm^3
h = latent heat of fusion, in joules per cm^3
$v_w = lA$ = volume of fuse element.
$v = lA_2$ = volume of melted part of fuse element.
R = resistance of circuit.
R_f = resistance of fuse element.
R_1 = resistance of fuse element at start of melting.
t = time.
t_f = pre-arcing time.
$T = L/R$ = time-constant.
L = inductance.
C = total current/heat integral.
l = length of fuse element.
A = cross-sectional area of fuse element.
A_1 = cross-sectional area of solid part of fuse element during melting.
A_2 = cross-sectional area of liquid part of fuse element during melting.
d = diameter of fuse element.
ρ = resistivity.
ρ_0 = resistivity at ambient temperature.
ρ_1 = resistivity (solid) at melting temperature.
ρ_2 = resistivity (liquid) at melting temperature.
ρ_m = harmonic mean of solid and liquid resistivities at melting temperature.
α = temperature coefficient of resistance (solid), ambient temperature as datum.
β = temperature coefficient of resistance (liquid), melting point as datum.
E = circuit e.m.f. (D.C.) or peak value of circuit e.m.f. (A.C.).
V_a = arc voltage of fuse.
k = Preece's coefficient.
W = peak value of inductive energy.

but if 60-amp. fuses are included in the circuit the current peak is reduced to 5 kA or only 6 per cent and the mechanical and thermal stresses, which are proportional to the square of current, are reduced to negligible proportions. The current-restricting effect also greatly reduces both damage at the point where the short-circuit takes place, and disturbance to the system.

(*b*) *Circuit-breakers.* Similar considerations arise in connection with circuit-breakers. The presence of the fuses may reduce the peak current to a value below the rating of the breaker which can therefore safely be closed on a short-circuit; existing components such as current transformers can be kept in service, but would have to be replaced if instead of adding fuses the circuit-breaker were replaced by one of higher short-circuit rating.

APPENDIX 1*

The Current/Heat Integral of a Fuse Element on Short-Circuit.

Heating of the fuse element to melting point.

Since energy loss is negligible, the rise of temperature of the element is given by

$$cv_w\frac{d\theta}{dt} = I_{ip}^2R$$

Introducing the dimensions and resistivity of the element,

$$clA\frac{d\theta}{dt} = I_{ip}^2\rho\frac{l}{A} \text{ or } c\frac{d\theta}{dt} = i^2\rho$$

This can be written as

$$\frac{c}{\rho}d\theta = i^2dt \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where the left-hand side contains, besides the temperature θ, only two properties of the fuse material while the right-hand side is purely the square of the current in the differential of time.

The resistivity of the metal can be assumed to increase linearly up to the melting temperature.

Thus

$$\rho = \rho_0(1 + \alpha\theta) \quad . \quad . \quad . \quad (5)$$

Integrating equation (4) and substituting in (5),

$$\frac{c}{\rho_0}\int\frac{d\theta}{1 + \alpha\theta} = \int i^2dt \quad . \quad . \quad . \quad (6)$$

* While lack of space prevents the author from enumerating comparisons between test-results and results calculated on the basis of the Appendices, he would point out that many such comparisons have been made and show a very satisfactory measure of agreement.

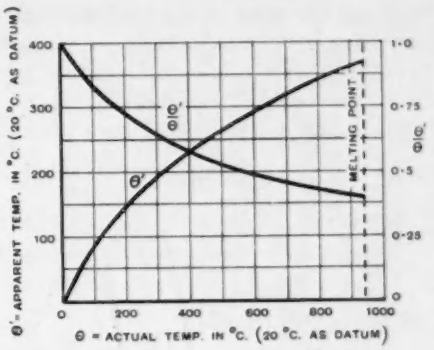

Fig. 25.—Relation between actual and apparent temperatures for silver fuse element up to melting point.

θ = actual temperature.
θ' = apparent temperature.

The right-hand side is the integral of the square of the current density during the time considered, while the integral on the left-hand side is a function only of temperature, which can be called the "apparent temperature."

Thus $$\theta' = \int\frac{d\theta}{1 + \alpha\theta} = \frac{\log_e(1 + \alpha\theta)}{\alpha} \quad . \quad (7)$$

which takes account of the variation in resistivity.

Fig. 25 shows the relation between apparent and real temperatures for silver, which has a mean temperature coefficient of 0·00442, referred to an ambient temperature of 20 degs. C.

As, according to equation (6),

$$\theta' = \frac{\rho_0}{c}\int i_2dt \quad . \quad . \quad . \quad (8)$$

it can be seen that θ' is the temperature which would be reached if the resistivity remained at its initial value ρ_0. The heating of the fuse wire for current varying in any way with time can therefore be found by determining the apparent temperature from equation (8) and then taking the real temperature from fig. 25 or equation (7).

From (8),

$$\int i_2dt = \frac{c}{\rho_0}\theta' = \frac{c}{\rho_0\alpha}\log_e(1 + \theta\alpha) \quad . \quad (9)$$

Melting of the fuse element.

It is assumed that a melted zone, starting from zero, gradually spreads till it covers the entire cross-section.

Thus $$hdv = I_{ip}^2Rdt \quad . \quad . \quad . \quad (10)$$

The resistance of a length l of element during fusion is

$$R = \frac{1}{\frac{A_1}{\rho_1 l} + \frac{A_2}{\rho_2 l}}$$

Now $$v_w - v = lA_1 \quad . \quad . \quad . \quad . \quad (11)$$

so that $$R = \frac{R_1}{1 - \frac{v}{v_w}\left(1 - \frac{\rho_1}{\rho_2}\right)}$$

From equations (10) and (11),

$$\frac{hdv}{v_w} lA = \frac{I_{fp}^2 \rho_1 l/A}{1 - \frac{v}{v_w}\left(1 - \frac{\rho_1}{\rho_2}\right)}$$

Integrating between the limits $v = 0$ and $v = v_w$,

$$\int i^2 dt = \frac{h}{2\rho_1}\left(1 + \frac{\rho_1}{\rho_2}\right)$$

Now $$\rho_m = \frac{2}{\frac{1}{\rho_1} + \frac{1}{\rho_2}} = \frac{2\rho_1}{1 + \frac{\rho_1}{\rho_2}}$$

The simple result is that

$$\int i_2 dt = \frac{h}{\rho_m} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Heating of the fuse element to vaporization point.

Again assuming a linear increase of resistivity with temperature,

$$\rho = \rho_2(1 + \beta\Delta\theta)$$

Thus if $\Delta\theta' = \frac{1}{\beta}\log_e(1 + \beta\Delta\theta)$, then from (4), in the same way as before,

$$\int i^2 dt = \frac{c}{\rho_2}\Delta\theta' = \frac{c}{\rho_2\beta}\log_e(1 + \beta\Delta\theta) \quad . \quad . \quad (13)$$

which gives the current/heat integral for heating in the liquid state, in the same way that equation (12) gives the current/heat integral for fusion and equation (9) the current/heat integral for heating in the solid state. All three are constants for a given metal.

The total current/heat integral is therefore given by

$$C = \frac{c}{\rho_0\alpha}\log_e(1 + \alpha\theta) + \frac{h}{\rho_m} + \frac{c}{\rho_2\beta}\log_e(1 + \beta\Delta\theta)$$

Calculations of C for silver, copper and zinc are given in Table 6, for an ambient temperature of 20° C.

APPENDIX 2*

Calculation of Peak Current in a Quick-Acting Fuse.

The rate of rise of current for direct current or symmetrical alternating current is given by E/L, denoted by B.

Then $i = (B/A)t$, so that

$$C = \int_0^{t_f} i_2 dt = \left(\frac{B}{A}\right)^2 \times \int_0^{t_f} t^2 dt$$

Solving, $$t_f = (3C)^{\frac{1}{3}} A^{\frac{1}{3}} B^{-\frac{1}{3}}$$

Substituting $I_f = Bt_f$, the following value of peak current is obtained :—

$$I_f = (3C)^{\frac{1}{3}} A^{\frac{2}{3}} (E/L)^{\frac{1}{3}}$$

Turning into practical units, this gives for silver peak current $I_f = 0{\cdot}463 \times \sqrt[3]{(\text{Rate of rise})}$

$\times$ (Total cross-section in sq. mils.)$^{\frac{2}{3}}$

Assuming that the wire is circular and that its minimum fusing current $I_c = kd^{3/2}$ by Preece's Law,

$$I_f = (3C)^{\frac{1}{3}}(E/L)^{\frac{1}{3}}(\tfrac{1}{4}\pi d^2)^{\frac{2}{3}}$$

and substituting for d the value given by $I_c = kd^{\frac{3}{2}}$

$$I_f = (\tfrac{1}{4}\pi)^{\frac{2}{3}}\{(3C)^{\frac{1}{3}}k^{-1}\}(E/L)^{\frac{1}{3}} I_c^{\frac{2}{3}}$$

The second term is a property only of the fuse-wire material and its enclosure, the third a property only of the circuit, and the fourth shows that the peak current is proportional to the 8/9th power of the minimum fusing current.

APPENDIX 3*

Calculation of Peak Current in a Slow-Acting Fuse.

(a) For direct current.

The rate of rise is given by

$$i = i_0(1 - e^{-t/T})$$

Thus $$C = \int_0^{t_f} i^2 dt = i_0^2 \int_0^{t_f} (1 - 2e^{-t/T} + e^{-2t/T}) dt$$

$$= i_0^2\{t_f - \tfrac{3}{2}T + T(2e^{-t_f/T} - \tfrac{1}{2}e^{-2t_f/T})\}$$

If t_f is very small compared with T, this gives the results set out in Appendix 2.

If t_f and T are of the same order, it is necessary to use a graphical solution from the above equation.

If t_f is very large compared with T the exponential terms vanish and therefore

$$C = i_0^2\left(t_f - \frac{3}{2}T\right)$$

and $t_f = C/i_0^2 + \frac{3}{2}T$. Thus the presence of inductance results in a time delay of $\frac{3}{2}T$.

(b) For alternating current.

The extreme cases of symmetrical and completely asymmetrical short-circuits are considered.

Symmetrical short-circuit.

$$i = i_0 \sin \omega t$$

* See footnote to Appendix 1.

therefore

$$C = \int_0^{t_f} i^2 dt = i_0^2 \int_0^{t_f} \sin^2 \omega t dt = i_0^2 \left(\frac{t_f}{2} - \frac{\sin 2\omega t_f}{4\omega} \right). \quad (14)$$

For long periods the first term preponderates, so that $C = (i_0/\sqrt{2})^2 t_f$ which corresponds to the D.C. non-inductive case since $i_0/\sqrt{2}$ is the R.M.S. current density.

The equation can be expanded into a series and gives

$$C = i_0^2(\tfrac{1}{3}\omega^2 t_f^3 - \tfrac{1}{15}\omega^4 t_f^5 + \ \ldots)$$

If the time is short, all but the first term in the bracket may be ignored, so that $C = \frac{1}{3} i_0^2 \omega^2 t_f^3$, which is similar to that already deduced for a quick-acting fuse. The pre-arcing time in this case is given by

$$t_f = (3C)^{\frac{1}{3}} (i_0 \omega)^{-\frac{2}{3}}$$

Asymmetrical short-circuit.

Ignoring the decrement,

$$i = i_0 (1 - \cos \omega t)$$

$$\text{Thus } C = \int_0^{t_f} i^2 dt = i_0^2 \int_0^{t_f} (1 - \cos \omega t)^2 dt$$

$$= i_0^2 \left(\frac{3}{2} t_f - \frac{2 \sin \omega t_f}{\omega} + \frac{\sin 2\omega t_f}{4\omega} \right). \quad (15)$$

For long periods, the first term preponderates, so that C is sensibly equal to $\frac{3}{2} i_0^2 t_f$, showing that the pre-arcing time is only one-third of that for the symmetrical case. It must, however, be noted that in this particular instance the assumption of negligible decrement is unjustifiable in practice.

For short times, the sine terms above are expanded in series up to the third order, giving

$$\int i^2 dt = \frac{i_0^2 \omega^4 t^5}{20}$$

The input is thus proportional to the fifth power of the time and at first develops very slowly. The pre-arcing time in this case is given by

$$t_f = (20C)^{\frac{1}{5}} i_0^{-\frac{2}{5}} \omega^{-\frac{2}{5}}$$

To find the condition for which the pre-arcing times on symmetrical and asymmetrical short-circuits are equal, a graphical method is convenient. Curves connecting t_f and $\frac{C}{i_0^2}$ for a frequency of 50 cycles per second are plotted from equations (14) and (15) and are found to intersect at values for t_f of 6·8 milliseconds and for $\frac{C}{i_0^2}$ of 4·1 milliseconds. For a silver fuse-wire this value of $\frac{C}{i_0^2}$ gives a prospective R.M.S. symmetrical current density of $2{\cdot}02 \times 10^6$ amperes per sq. in.

APPENDIX 4*

Mathematical Analysis of Arcing Conditions.

In all the following a constant arc voltage is assumed.

TABLE 6.

All the following quantities are in the usual C.G.S. units except where otherwise stated in the List of Symbols.

		Silver	Copper	Zinc
ρ_0	=	1.63×10^{-6}	1.724×10^{-6}	6.20×10^{-6}
ρ_1	=	8.4×10^{-6}	10.2×10^{-6}	17×10^{-6}
ρ_2	=	16.6×10^{-6}	21.3×10^{-6}	37.3×10^{-6}
α (mean value)	=	0.00442	0.00439	0.00410
β (mean value)	=	0.00071	0.00038	−0.00010
θ_m	=	940° C.	1,063° C.	399° C.
θ_v	=	1,930° C.	2,280° C.	887° C.
$\Delta\theta$	=	990° C.	1,217° C.	488° C.
Specific heat (mean value)	=	0.059	0.101	0.097
c	=	2.595	3.76	2.88
Latent heat of fusion (mean value)	=	25.9	49.3	26.6
h	=	1,139	1,833	790
ρ_m	=	11.15×10^{-6}	13.79×10^{-6}	23.36×10^{-6}
θ'	=	371	396	236
$\Delta\theta'$	=	750	999	500
From the above figures, the following values for the current/heat integral are obtained :—				
First heating period, up to melting, according to (9)		5.91×10^8	8.63×10^8	1.096×10^8
Melting period, according to (12)		1.02×10^8	1.33×10^8	0.338×10^8
Second heating period, up to vaporization, according to (13)		1.07×10^8	1.76×10^8	0.386×10^8
Total current/heat integral, C		8.00×10^8	11.72×10^8	1.82×10^8

* See footnote to Appendix 1.

Calculation of current in arcing period for alternating current.

The case of a short-circuit at very low power factor will be considered.

Since resistance may be ignored,

$$\text{Circuit e.m.f.} = -LdI_{ts}/dt$$

At the instant t_f, the arc voltage V_a is introduced.

Then $$-LdI_{ts}dt + V_a + dI_{ta}/dt = 0$$

or $$LdI_{ts} = V_a dt + LdI_{ta}$$

Integrating, $LI_{ts} = V_a t + LI_{ta} + \text{constant}$ (16)

When arcing commences at the instant t_f, $I_{ta} = I_f$.

Therefore $LI_f = V_a t_f + LI_f + \text{constant}$,

or $$\text{constant} = -V_a t_f$$

so that, from (16) $LI_{ts} = V_a t + LI_{ta} - V_a t_f$

or $$I_{ta} = I_{ts} - \frac{V_a}{L}(t - t_f) \quad . \quad . \quad . \quad (17)$$

i.e., arc current = prospective current $-(V_a/L)$ (arcing time).

This result can be used to obtain the arcing time by substituting $I_{ta} = 0$ and expressing I_{ts} in terms of t.

A graphical construction derived from the above is given in Section (5) of the paper.

The relation between arc voltage and peak system-voltage can now be considered. The maximum slope of the prospective current curve is E/L. The slope of line X [fig.9(a)] is V_a/L. It will be seen from figs. 9(*a*) to 9(*d*) inclusive that unless the slope of line X is greater than the maximum slope of the prospective current curve, the arcing time may be very protracted under certain phase conditions. This can also be seen from the following, taking a symmetrical short-circuit by way of example.

From (17) $$I_{ta} = \frac{E}{\omega L}\sin\omega t - \frac{V_a}{L}(t - t_f)$$

Thus $dI_{ta}/dt = (E\cos\omega t - V_a)/L$, of which the maximum (arithmetical) value is $(E + V_a)/L$. If $V_a = E$, then the maximum value of dI_{ta}/dt is equal to twice the maximum value of dI_{ts}/dt; if $V_a = 2E$, then the corresponding maximum rate is 3 times.

Calculation of current and arc energy for direct current.

The voltage equation is

$$E + LdI_{ta}/dt + V_a + RI_{ta} = 0$$

Let $F = E + V_a$. This quantity is the arithmetical *difference* of the two voltages, and must be positive if extinction is to take place.

Then $$LdI_{ta}/dt = -(F + RI_{ta})$$

Integrating and putting $I_{ta} = I_f$ when $t = t_f$

$$\frac{L}{R}\log_e\frac{F + RI_{ta}}{F + RI_f} = -(t - t_f)$$

To determine the arcing time, $I_{ta} = 0$ is substituted in the above, giving

$$\text{Arcing time} = \frac{L}{R}\log_e\left(1 + \frac{RI_f}{F}\right)$$

and arc ampere-seconds $$= -\frac{L}{R}\int_{I_f}^{0}\log_e\frac{F + RI_{ta}}{F + RI_f}dI_{ta}$$

$$= \frac{L}{R}\left(I_f - \frac{F}{R}\log_e\frac{F + RI_f}{F}\right)$$

and arc energy $$= \frac{V_a L}{R}\left(I_f - \frac{F}{R}\log_e\frac{F + RI_f}{F}\right)$$

Expanding the above,

$$\text{Arc energy} = \frac{V_a LI_f^2}{2F}\left\{1 - \frac{2}{3}\frac{RI_f}{F} + \frac{1}{2}\left(\frac{RI_f}{F}\right)^2 - \ldots\right\}$$

If the fuse is quick-acting and the arc voltage considerably higher than the line voltage, then

Arc energy $= \frac{1}{2}LI_f^2 V_a/F$ approximately, and

$$\text{Arc energy ratio} = \frac{\frac{1}{2}LI_f^2 V_a/F}{\frac{1}{2}LI_f^2} \text{ approximately}$$

$$= \frac{V_a}{V_a - E} \text{ approximately}$$

Thus the arc energy ratio is independent of inductance; it approaches unity only when the arc voltage is much higher than the line voltage, and is then approximately independent of resistance also.

APPENDIX 5

Test Equipment.

The equipment used to obtain many of the test results quoted consists of:

(*a*) A high-power testing station[19] equipped with an alternator having a short-circuit rating of approximately 2,000 MVA and arranged for testing at any voltage between about 76 kV single-phase and 250 volts single-phase. The measuring equipment includes a magnetic oscillograph and 2 cathode-ray oscillographs.

(*b*) A D.C. testing station[29] equipped with two motor-generator sets, each set having 2 D.C. machines of a continuous rating of 1,270 kW, 900 volts. Provision is made for using 1, 2, or 4 of the D.C. machines in any desired connection. By using 4 machines in parallel a prospective current of approximately 60 kA at 750 V is available, and, using 2 machines in series, about 35 kA at 1,900 V. Reactors and resistors are available for controlling the circuit constants, and a 6-element magnetic oscillograph is used for the measurements.

In addition, a 250-kW, 600-volt D.C. motor-generator set and smaller sets were used for other tests.

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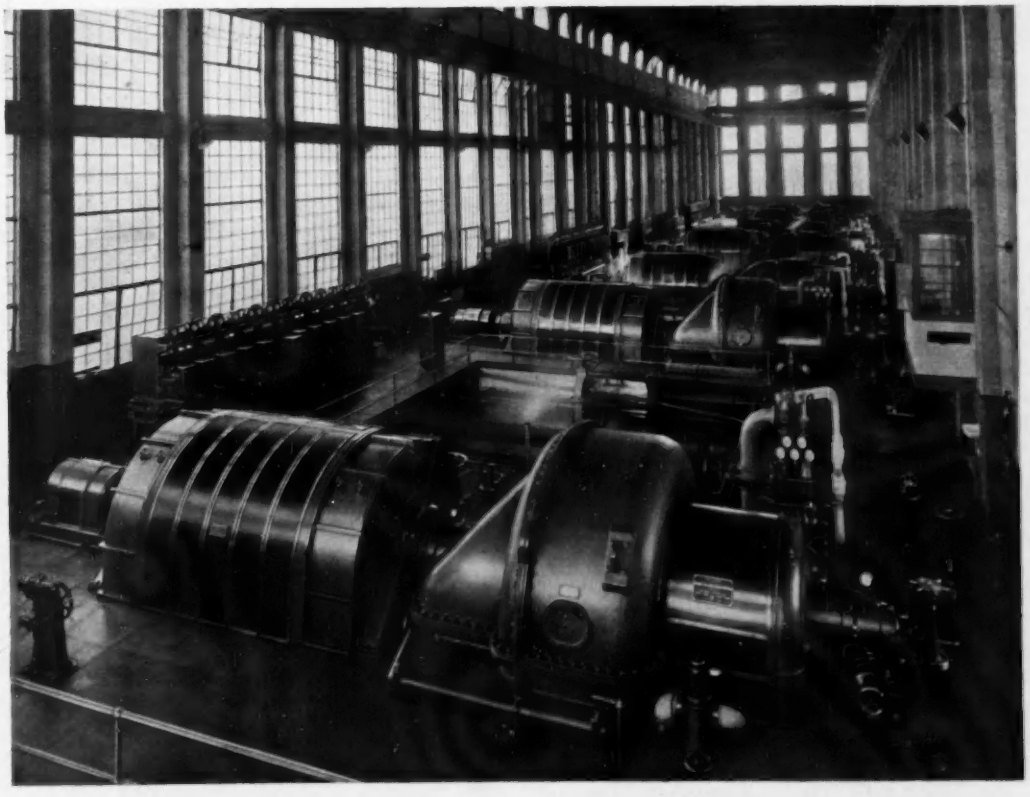
Turbo-alternators for the Municipality of Johannesburg.



The seven G.E.C. turbo-alternators shown are installed in the Jeppe Street power station of the Municipality of Johannesburg, and are designed for voltages of 20.5 kV and 6.6 kV. All the sets run at 3,000 r.p.m. and have an aggregate output of 110,000 kW; the generators feed on to the station busbars through suitable transformers. Further generating plant comprising three 37,500 kVA, 11 kV turbo-alternators, together with a 2,500 kW, 6,000 volt house set, are being supplied for the equipment of another of the municipal power stations.

Outdoor Substations in Madras.

By J. MEEK, B.Sc.,
G.E.C. Resident Engineer, Madras.

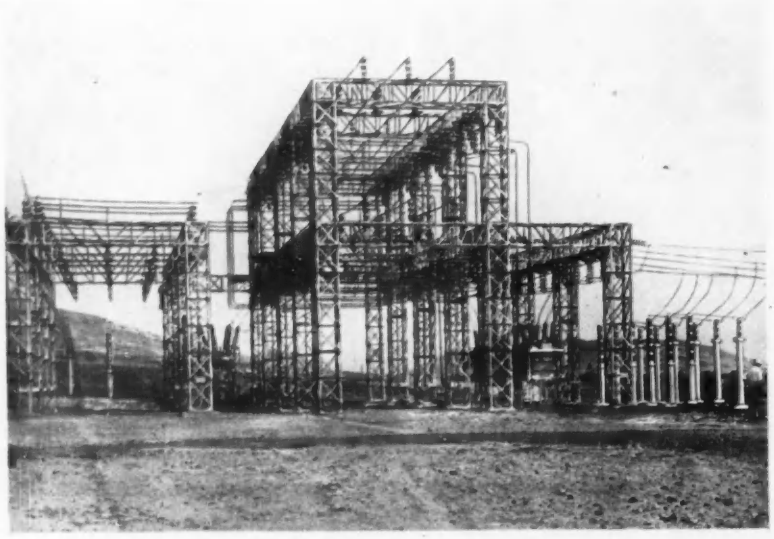


Fig. 1.—Madura Substation, showing the double busbar circuit, the 66 kV oil circuit breakers, and the air break switchgear.

INTRODUCTORY.

THE Madras Presidency covers an area in South India of 142,000 square miles and supports a population of about 48 millions. An idea of the size may be gained by comparison with that of England which has an area of 50,000 square miles and a population of about 38 millions.

The people in the Madras Presidency are mainly engaged in agriculture and most of them live in villages. The capital, Madras City, has a population now approaching 800,000, but the next biggest town, Madura, has a population under 250,000. Four main languages are spoken—Tamil, Telugu, Canarese and Malayalam—and efforts are now being made by the Congress Government to introduce Hindi with the ultimate object of providing India with a common language. English is widely spoken amongst the more educated classes.

Some idea of the climate of Madras can be obtained from the following significant clause which usually appears in the Madras Government Electricity Department's specifications :

> "The temperature in the shade will vary from 50 degs. F. to 110 degs. F. The maximum temperature in the sun may be assumed to be 150 degs. F. The relative humidity will vary from 60 to 80 per cent."

Although there is very little rain in most parts of the Presidency for the greater part of the year, there is considerable rainfall during the south-west and north-east monsoons, especially on the Eastern and Western Ghats where over 400 ins. is registered in some places.

ELECTRICITY SUPPLY.

Until comparatively recently the only supply system of any importance was that of the Madras Electric Supply Corporation which has held a licence from the Government for the generation and distribution of electricity in Madras City and a surrounding area for the last thirty-one years. This Corporation has a steam-turbine power station with an installed capacity to-day of over 53,000 kVA. A number of other licences have been worked in various parts of the Presidency using Diesel engine plant.

The Presidency has no coal or oil deposits but is fairly well provided with water power in the south. In 1924 the Madras Government decided to take charge of all hydro-electric surveys and projects and bought back concessions previously granted to private concerns which had not been developed.

The Pykara Scheme in the Nilgiri Hills was taken up first and a supply given to Coimbatore and other places early in 1933. The Pykara and Mettur Power Houses are situated at a distance of some 280 and 200 miles respectively to the south-west of Madras. Pykara is a high head scheme with storage, a fall of 3,074 ft. being utilised. The total capacity of the plant installed at the Pykara Power House exceeds 48,000 kVA.

When the great Mettur Dam was built some years ago for irrigation control, hydro-electric pipes were left ready in the Dam. This development was the next to be taken up and the station commenced supply in 1937. The present installed capacity of plant is 37,500 kVA operating on a head which varies from 60 to 160 ft.

The Madras Government have now approved of a further development, the Papanasam Hydro-Electric Scheme, which is at present under construction. Later it is proposed to operate the Papanasam Hydro Station in conjunction with a Thermal Station. A number of other hydro-electric

schemes have been investigated by the Madras Government and are available for development when required.

In addition to these schemes the Madras Government have provided two steam stations in the north, one at Vizagapatam and one at Bezwada. Hydro-electric stations may be built in the future if the load developed warrants the expenditure.

The Pykara System is linked with the Mettur System at Erode, and Papanasam will in due course be connected into both the Pykara and Mettur Systems at Madura. There is no connection between the hydro-electric systems in the south and the two thermal schemes in the north. The Madras Grid therefore consists of the network of high tension transmission lines and substations fed from Pykara and Mettur to be joined in the near future by Papanasam. The extent of this Grid may be judged from the detailed information given below.

Pykara System.

A double-circuit line operating at 110 kV runs from Pykara to Coimbatore 49 miles away. From Coimbatore to Erode there is a double-circuit line 58 miles long and another to Madura, 125 miles in length, both operating at 66 kV.

Mettur System.

Mettur is connected to Erode by a double-circuit 66 kV line 36 miles long. The double-circuit line from Mettur to Singarappet Switching Station is 64 miles in length. This line is operating at 66 kV but will eventually be changed over to 110 kV, as it is ultimately intended to build a line from Singarappet to Madras, a distance of about 130 miles, in order to link up with the thermal station there.

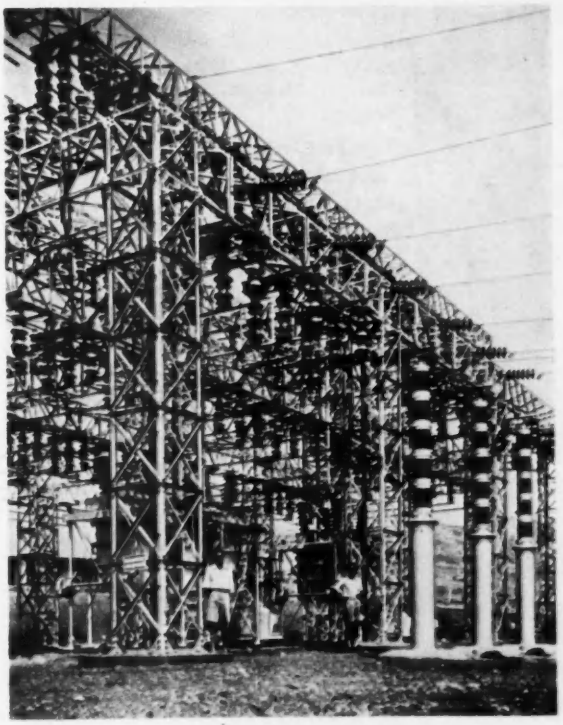


Fig. 2.—Madura Substation, the incoming bays.

From Singarappet there is a single-circuit 66 kV line to Vellore 72 miles away. There is also a 66 kV double-circuit line from Erode to Tanjore 114 miles in length, and a single-circuit 66 kV line is continued to Negapatam a further 49 miles away. These figures indicate the extent and rapid growth of the Grid.

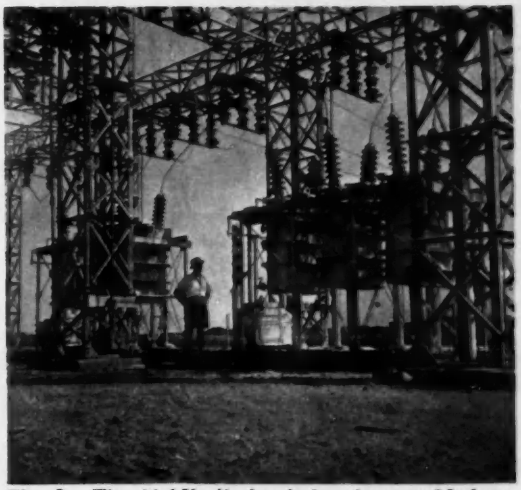


Fig. 3.—The 66 kV oil circuit breakers at Madura Substation.

In addition to the lines mentioned there are many miles of feeder lines and also a considerable mileage of 33 kV and 11 kV transmission in connection with the Vizagapatam and Bezwada Schemes. The length of these transmission lines has necessitated the introduction of synchronous condensers at Trichinopoly and Madura.

DEMAND FOR POWER.

The demand for power has greatly exceeded the most optimistic estimates, and additional plant and extensions to lines and substations have had to be installed several years in advance of the original programme. Not only have existing textile mills and other works changed over from their own power plant to the Grid, but many new mills have come into being as a direct result of the Government's initiative in hydro-electric development. Local authorities and private companies holding licences from Government have closed down their Diesel plant when the Grid brought hydro-electric power to their town or district.

The Grid supplies power direct to many tea factories, ginning factories, oil mills, chemical works and even to farmers in outlying districts. Over a 1,000 small pumping sets are now connected to the systems for pumping water from wells into the fields. The ryot has found it cheaper and more convenient to water his fields by means of electricity than by

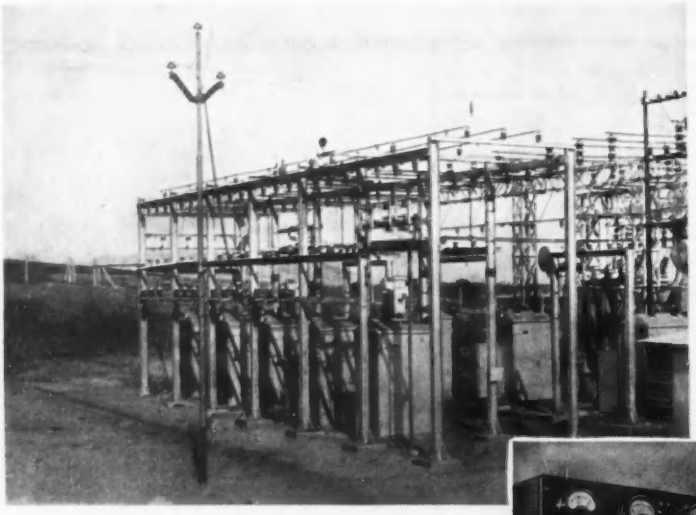


Fig. 4.—11 kV substation at Madura, showing metering and oil switch kiosks together with air break switchgear and multigap type lightning arresters, with the 11 kV/440 volt auxiliary lighting transformers.

using bullocks as his forefathers have been doing for centuries.

Substations.

The General Electric Co., Ltd., has supplied and erected a large number of the outdoor substations on the Pykara and Mettur System as well as on the Bezwada and Vizagapatam Schemes. The equipment supplied includes outdoor type oil switches, air break switches and isolators, busbar supports and connections, low tension lightning arresters, metering kiosks, control boards, together with the necessary structural steelwork.

PYKARA AND METTUR SYSTEM.
Madura 66 kV Receiving Station.

The Madura receiving station is one of the most important on the Grid. At present it is fed by the double-circuit line from Pykara by way of Coimbatore, but it will in due course be connected with the Mettur System, the Papanasam System and with the Periyar Scheme, if that is developed.

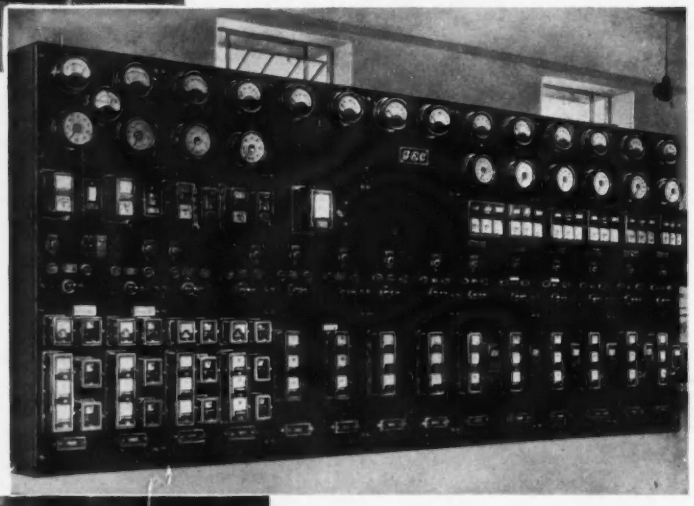


Fig. 5.—Madura Substation, the 66 kV and 11 kV control panels.

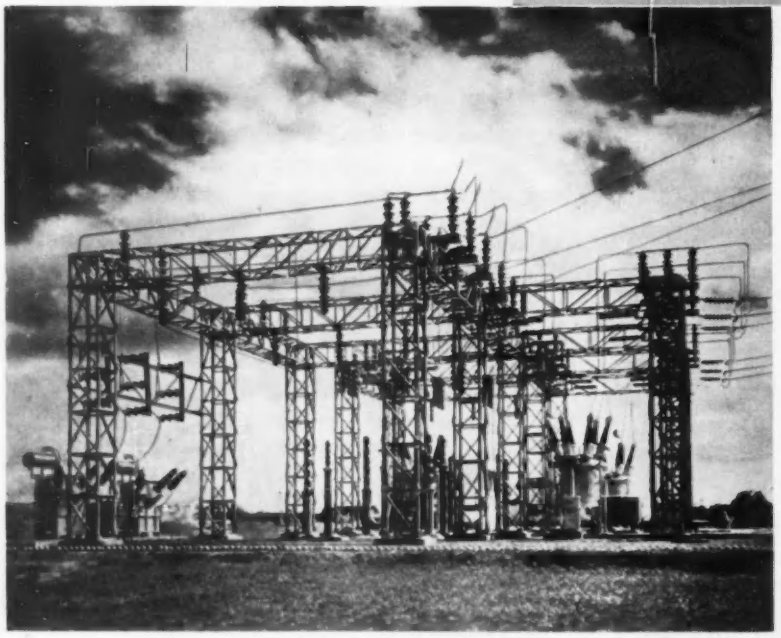


Fig. 6.—66 kV substation at Singarappet, showing the air break switchgear and transformers.

The equipment on the 66 kV incoming side comprises a double circuit busbar in tier formation; consisting of copper-clad steel tubes supported on three-unit post insulators fitted with expansion joints. Fig. 1 is a general view of the station and shows the double busbar circuit together with the 66 kV oil circuit breakers and the air break switchgear, while fig. 2 is a nearer view of the incoming bays.

Both the incoming and outgoing circuits are controlled by rotary type isolators and oil circuit breakers (fig. 3). The transformers are interconnected by a separate set of busbars of similar construction to the main bars. The transformer busbars are connected to the power transformer by vertical horn air break switches. From the double

bus circuit a potential transformer connection is taken through rotary isolators, limiting resistances and Quenchol fuses; one potential transformer being used for both sets of bars.

From the low tension side of the transformer connections are taken through metering kiosks to the 11 kV busbars. The 11 kV feeder circuits from these bars are controlled by kiosks equipped with isolators and oil circuit breakers. Thence the supply is taken to the overhead outgoing lines, each of which is protected from lightning storms by multigap type lightning arresters (fig. 4).

There is a small transformer, fed from the 11 kV bars, which supplies current for local lighting and is protected by lift-out Quenchol fuses.

The control panels for both the 66 kV and 11 kV switchgear, seen in fig. 5, are accommodated in a nearby control house and are equipped with oil switch controllers, instrument and protective relays and serve to control the main incoming and outdoor high tension circuit breakers, the transformers, and the oil circuit breakers on the low tension side.

The two outdoor oil circuit breakers on the 66 kV side have a breaking capacity of 750 MVA. The construction is such that for each phase a separate switch body and tank are provided, the whole being mounted together in a strongly constructed tubular steel framework. The operating mechanism controls all three switches in unison. The closing and tripping solenoids in the mechanism ease are remote electrically controlled, but provision is made locally for manual operation. The breakers are fitted with arc control pots which ensure satisfactory operation over the entire range of breaking currents. A tank lowering device is provided to facilitate inspection.

Fig. 7.—Two 100 kVA 66 kV/110-400 volt transformers protected by pole operated isolators at Singarappet Substation.

The vertical air break switches are capable of breaking a heavy charging current without deterioration of the contacts by virtue of the arc being carried upwards and away from them and eventually broken on the horns a good distance from the contacts. The fixed contacts are self-aligning and are designed to carry a considerable overload without overheating. There are 3 three-unit insulators per phase, one of which is mounted on a heavy bearing unit with two taper-roller thrust bearings. When the switch is operated this insulator rotates and carries a crank which pulls the blade into the "off" position. Each phase is built up separately on a channel iron base.

Kattabettu Substation.

The Kattabettu substation is situated some 29 miles from the Pykara Power House, and is the

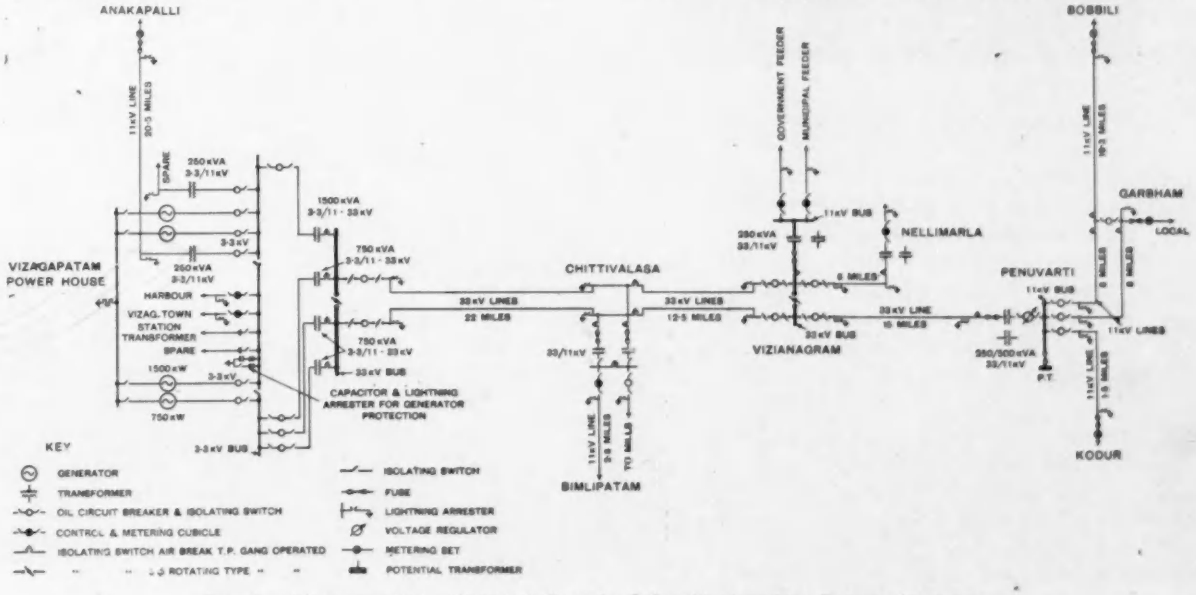


Fig. 8.—Diagram of the electrical layout of the Vizagapatam Thermal Scheme.

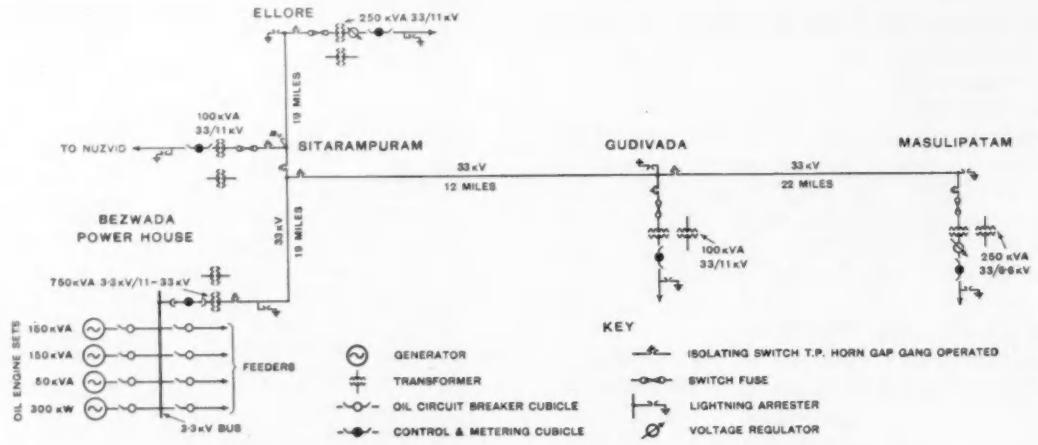


Fig. 9.—Diagram of the electrical layout of the Bezwada Thermal Scheme.

transformer switching station supplying Mettupalaiyam and Karamadai. The supply is tapped off from one of the double-circuit transmission lines running

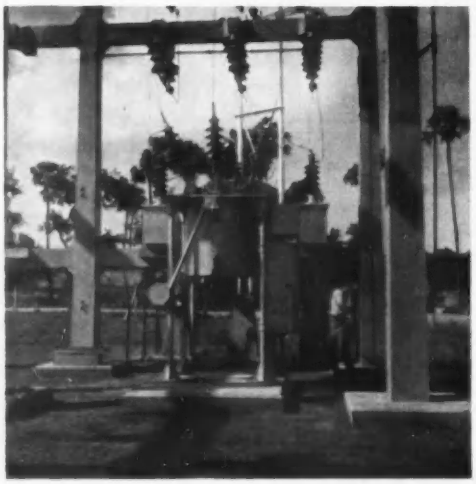


Fig. 10.—33 kV oil circuit breaker in the substation at Vizianagram.

between Pykara Power House and Coimbatore, which when originally installed operated at 66 kV but were later raised to 110 kV.

The switching is so arranged that two triple-pole sets of 110 kV rotary isolators are mounted in series in the transmission line run, the feeder supply being taken from the centre point of these two isolators, thus enabling a supply to be taken either from Pykara or Coimbatore.

The transformer, which is rated at 250 kVA 110kV/66kV/11kV, is protected on the high tension side by means of a 110 kV vertical horn, air break switch and 110 kV Quenchol fuses, and on the low tension side by the usual isolators and oil circuit breakers.

Singarappet Substation.

This substation, a general view of which is seen in fig. 6, is insulated for a working voltage of 110 kV, but is at present being used as a 66 kV terminal switching station. There are two incoming lines from the Mettur Power House, approximately 64 miles long, which are controlled by line isolators fitted with earthing switches, and by suitable outdoor type oil circuit breakers. The incoming lines feed a set of busbars, the two lines being split on the bars by means of bus section isolators and oil circuit breakers thus enabling the lines to run singly or in parallel.

From the busbars a supply is taken in the first place to a chain of transforming substations to the north, in the direction of Madras. This transmission line is controlled by a horn air break switch with earthing switch and gives a supply to Tiruppattur,

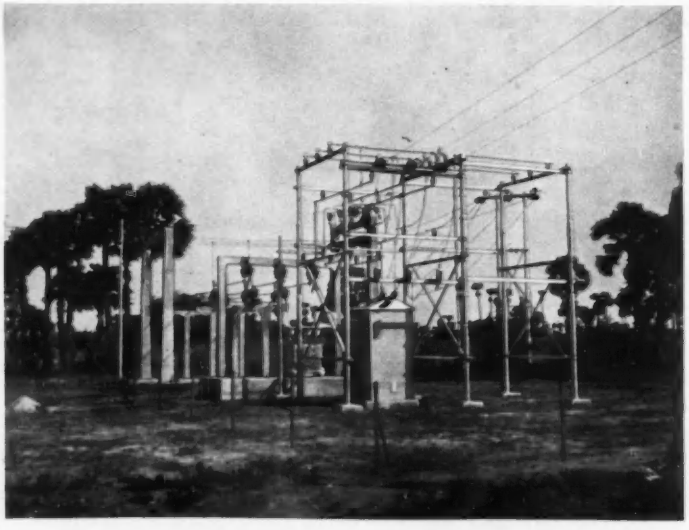


Fig. 11.—Chittivalasa Substation.

which is the first station in the chain, thence it proceeds to Vadakapetti and Ambur.

A further supply is taken from Singarappet for local use and is stepped down by a 100 kVA 66 kV/ 110-400 volt transformer which is protected by pole operated isolators in conjunction with Quenchol fuses (fig. 7).

Since the original switching station was installed an additional feeder has been added giving a supply to Karimangalam at 33 kV which is taken from the

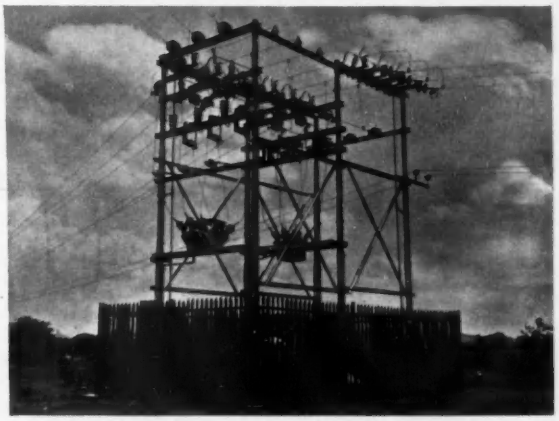


Fig. 12.—Garbham 11 kV pole type switching station.

main busbar through horn air break switches to a 500 kVA 66 kV/33 kV-11 kV step-down transformer. The 33 kV side is controlled by rotary type isolators and an oil circuit breaker, while the outgoing 33 kV transmission line is protected by multi-gap lightning arresters.

Tiruppattur Substation.

There are a considerable number of points where a few hundred kilowatts have to be tapped off from a single-circuit 66 kV line for local distribution. Tiruppattur substation on the Mettur System is a case in point.

The 66 kV transmission lines are brought through this station, the outgoing side being controlled by a sectionalising horn air break switch equipped with an earthing device. The lines are tapped on the incoming side, the supply being taken through a horn air break switch and a set of 66 kV Quenchol fuses to a step-down transformer. A spare transformer is kept disconnected but ready for easy replacement when necessary.

The local supply from the low tension side of the transformer, which is at 11 kV, is brought out to a set of busbars and is then taken through metering kiosks to the various feeders which are protected by lightning arresters.

The kiosks are constructed of sheet steel, the 11 kV line being brought in through bushings in the roof. The equipment includes a hand operated oil circuit breaker of 50,000 kVA breaking capacity, a Trivector meter for indicating the power factor of the system, together with the necessary potential and current transformers for operating the instruments. The tripping circuit for the overload relays is supplied from an alkaline battery provided with a metal rectifier unit for trickle charging.

A 10kVA 11kV/400-230 volt transformer protected on the 11kV side by Quenchol rewireable fuses supplies lighting in the attendants' and patrolmen's quarters and power for a portable oil filter, when required, for the main transformer. An outdoor distribution fuse box is provided adjacent to the auxiliary transformer. A recording voltmeter and an alarm bell are installed in the attendants' quarters ; a double-throw switch enables the attendant to put the voltmeter on to the potential transformer of either kiosk. The bell indicates when a circuit breaker has tripped.

There are two further stations at Ambur and Vadakattappati which are similarly equipped to Tiruppatur.

VIZAGAPATAM AND BEZWADA SCHEMES.

The Vizagapatam and Bezwada thermal stations are situated in the north of the Presidency, and diagrams of the electrical layout of the systems supplied from these stations are shown in Figs. 8 and 9, respectively.

Fig. 13.—General view of the 33kV/11kV Penuvarti Substation.

Vizianagram and Chittivalasa Substations.

Vizianagram is one of the three 33 kV/11 kV substations on the Vizagapatam Thermal System, and is arranged for extension at a later date.

The main 33 kV structure is designed in reinforced concrete while the 11 kV side is of tubular steel construction. The incoming single circuit 33 kV line from the Power House at Vizagapatam is taken through a rotating type isolator on the main busbars, lightning protection being provided by 33 kV multigap arresters. The outgoing line to Penuvarti, Bobbili and the Manganese Mines is controlled by a hand operated oil circuit breaker with rotating type isolating switches on either side of it.

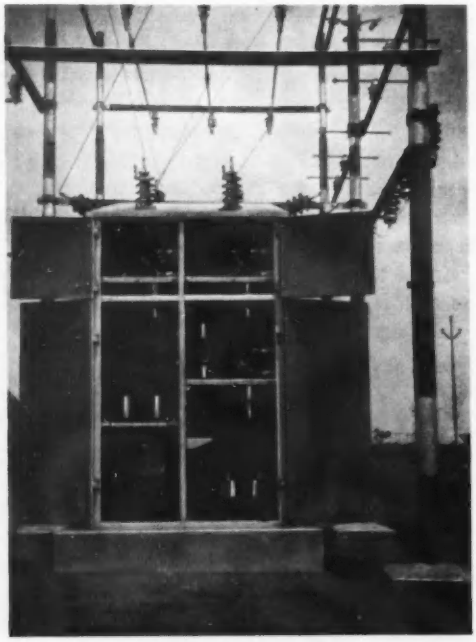


Fig. 14.—11 kV switching kiosk at Arkonam Substation showing the oil circuit breaker, potential transformer and gang operated isolators.

The three-phase oil circuit breaker is compactly arranged in a cylindrical tank with ample electrical clearance and is designed for a breaking capacity of 250,000 kVA (fig. 10). The incoming and outgoing lines are taken through condenser type bushings on the domed top cover. A control chamber, rigidly clamped to the tubular uprights which support the breaker, houses the operating mechanism, trip gear, fuses and auxiliary switches. Side-blast arc control devices are fitted for rapid extinction of the arc. A manually operated hoist is provided for lowering and raising the tank.

The main transformer is protected by a set of 33 kV Quenchol rewireable fuses.

From the 11 kV busbars connections are taken to two outdoor cubicles, similar to those already described, which house the 11 kV oil circuit breaker and metering equipment. Multigap arresters are provided for the protection of the outgoing feeders against lightning. A recording voltmeter and an alarm bell are installed in the attendant's quarters.

At a similar substation at Chittivalasa (fig. 11) one of the 11 kV switches is fitted with an auto-reclosing device, which consists essentially of a catching and re-setting gear operated by a falling weight and a mechanical delay action to time the instant of reclosing. All the main components, with the exception of the weight, are mounted in a case on the front of the cubicle directly above the circuit breaker closing and tripping levers. The reclosing gear is automatically locked out after the breaker has been reclosed four times. An indicator shows the number of times reclosing has taken place.

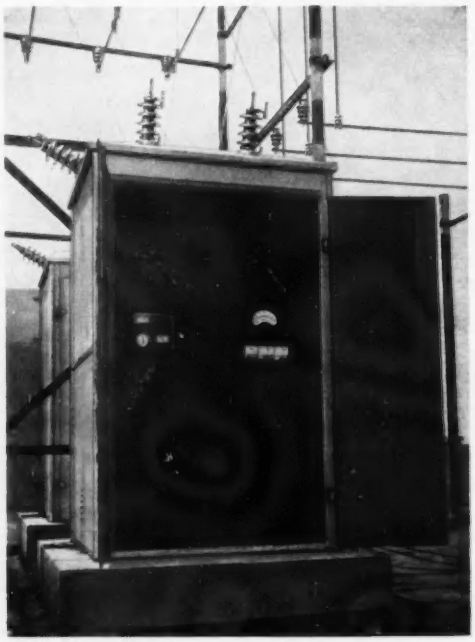


Fig. 15.—External view of the 11 kV switching kiosk at Arkonam Substation.

Garbham Substation.

This is an 11 kV substation on the Vizagapatam Thermal System, designed essentially for metering and with suitable provision for taking off a local supply. The station comprises a structure assembled from tubular steel poles mounting two gang operated air break switches controlling the double-circuit incoming line from Penuvarti and a similar switch controlling an outgoing line to Bobbili. Fig. 12 shows the substation at Garbham with the oil circuit breaker, metering equipment and lightning arresters, while fig. 13 is a general view of the 33 kV/11 kV outdoor substation at Penuvarti. In addition, the equipment of the Garbham substation includes an 11 kV interconnecting oil circuit breaker of 50,000

MVA breaking capacity and with direct acting overload trips, and a totally enclosed metering equipment. Pole operated single pole isolating switches serve to isolate the oil circuit breaker which is mounted on the structure and is remote mechanically operated from ground level.

The two current transformers and the potential transformer for the operation of the metering equipment are housed in a common oil filled tank into which the 11 kV connections are taken through concentric bushings. A Trivector meter and an electrically wound time switch are housed in a separate weatherproof box. Typical examples of 11 kV switching kiosks, showing the oil circuit breaker, gang operated isolators and potential transformer, are seen in figs. 14 and 15 which give two views of the kiosk at Arkonam substation while in course of construction.

Masulipatam Substation.

This is a 33 kV/6·6 kV terminal substation on the Bezwada Thermal System with a four-leg A-type reinforced concrete structure for the 33 kV busbars, an air break switch, and 33 kV multigap lightning arresters. The 11 kV structure and kiosks follow the standard lines already described.

CONCLUSION.

The development of the various schemes described has been under the direction of Sir Henry Howard, C.I.E., M.C., M.I.E.E., to whom the author is indebted for permission to publish the foregoing information and illustrations.

Lighting in the Board Room of the Milk Marketing Board.

The lighting in this Board Room is by means of standard Osram lamps enclosed in pendent fittings of white opal glassware and brass metalwork, finished in bronze, and designed to give ample illumination with a complete absence of glare. Special lighting fixtures equipped with Osram lamps are also used in the entrance hall, the corridors, the cafeteria and the executive dining room. Additional illumination in the cafeteria and entrance hall is provided by means of "Osira" fluorescent tubing.

Wave Guides.

By J. E. HOULDIN, Ph.D., B.Eng.,

G.E.C. Research Laboratories.

Following a brief historical introduction, the properties and characteristics of hollow metal tubes for the transmission of frequencies above 1000 Mc/sec. are dealt with in detail. The patterns of electromagnetic energy radiated from open ended tubes are discussed and shown to be of exceptional interest at frequencies above 600 Mc/sec. In conclusion a number of experimental results obtained by the author are quoted.

This paper was read before the Students' Section of the Institution of Electrical Engineers in February, 1940.

1. BRIEF HISTORICAL INTRODUCTION.

IN 1897 Lord Rayleigh pointed out and proved mathematically that electromagnetic waves could be propagated along hollow metal tubes. This novel form of transmission could not at that time be utilised as the cross-sectional dimensions of the required tube are of the same order as the wavelength of the waves to be transmitted. Because of this limiting factor and the lack of suitable apparatus to generate very high frequencies, the use of the hollow metal tube for the propagation of energy attracted little attention until 1934.

The introduction of positive-grid triode and magnetron oscillators made it possible to generate electrical oscillations of frequencies up to 3000 Mc/sec.

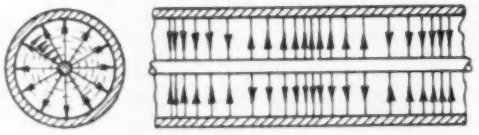


Fig. 1.—Approximate configuration of lines of electric and magnetic force in a coaxial conductor.
Lines of electric force ——>——
Lines of magnetic force - - - > - - -

The interest in these high frequencies and their applications correspondingly increased. In 1935 two experimental and theoretical studies of the hollow metal tube as a transmission line at very high frequencies were carried out independently in America.

Since this date the use of the hollow metal tube for the propagation of energy has received extensive study in America and France. The field of application of the system has been increased further by the study of the hollow metal tube as a radiator or receiver of electromagnetic energy. This latter application would seem to be that in which the hollow metal tube will be first used extensively.

The hollow metal tube may have one of many possible cross-sections—circular, rectangular, etc.—and for each cross-section there will be a corresponding system of waves within the tube. It would be impossible to cover the whole ground in one paper, so that it has been decided to divide the present paper into two sections. The first will deal with the use of a cylindrical hollow metal tube as a transmission line for electromagnetic energy. The second portion of the paper will be a discussion of the use of a rectangular hollow metal tube as a radiator or receiver of energy. In this way a large amount of the possible field will be covered and possible confusion will be eliminated.

2. HOLLOW METAL TUBES AS TRANSMISSION LINES.

(*a*) *Nature of Problem.* The transmission system consists of a hollow metal tube of circular cross-section, filled with a suitable dielectric of low losses, and terminal devices for connecting the tube to the conventional two-conductor receiving and transmitting apparatus. It has been shown by analysis that many types of electromagnetic waves can exist in the metal tube. Only four of these types merit special consideration.

Electromagnetic waves can travel in free space or along a co-axial cable or parallel wire system. In many ways the waves existing in hollow metal tubes differ considerably from either of these more usual types of wave.

In order to introduce the subject it has been deemed advisable to compare the electromagnetic waves travelling along the hollow tube with those along a co-axial cable system.

(*b*) *Field Orientation.* When dealing with two-conductor circuits it is usual to resort to the concept of current electricity. In the case of the hollow metal tube, it is essential to favour the concept of electric and magnetic lines of force. This concept

can of course be applied to any problem dealing with the transfer of electromagnetic energy, and in fig. 1 it is shown applied to the co-axial cable at any arbitrary instant of time. The magnetic force is constant around any circumference, whilst the electric flux is constant across any sector. There are

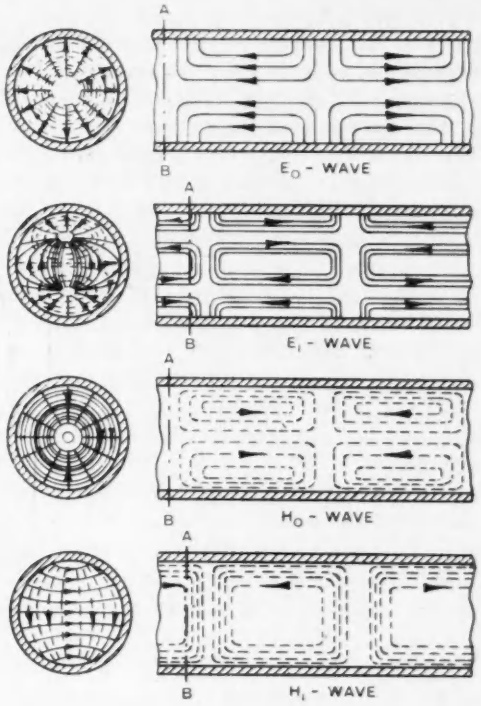


Fig. 2.—Approximate configuration of lines of electric and magnetic force in a cylindrical hollow metal tube excited by the four main types of waves.
Lines of electric force ——>——
Lines of magnetic force - - - > - - -

no components of electric or magnetic force in the direction of propagation of energy.

The field orientations of the four main types of waves propagated along a cylindrical hollow metal tube are shown in fig. 2. In the upper two sketches it will be seen that there is a component of electric force along the axis of the tube in the direction of propagation. There is no magnetic force component in this direction. For this reason the waves are designated *E*-waves. The suffixes serve to distinguish the two types of *E*-waves. Similarly in the two lower sketches there is a component of magnetic force along the axis but no electric force component, and the waves are designated *H*-waves.

Here immediately the presence of axial components of electric or magnetic force gives a striking difference between the co-axial and the hollow tube types of waves.

(*c*) *Frequency Limitation.* Owing to the fact that there is no component of electric or magnetic force along the axis of propagation, a co-axial cable is able to transmit a wave of any frequency. If an axial component is present, as in hollow tube transmission, boundary conditions for the magnetic or electric force have to be satisfied. In analysis, this means that a hollow tube will only transmit frequencies greater than a certain critical frequency. This frequency depends upon the diameter of the tube, the material within the tube, and in general, the type of wave being transmitted. The critical frequencies and their corresponding free space wavelengths for the four main types of waves are shown in the accompanying table. If the tube is filled with air ($K = 1$), the critical wavelength is of the same order as the diameter of the tube.

(*d*) *Velocity of Propagation.* The phase velocity of propagation of electromagnetic waves along a concentric cable, and hence their wavelength, depends mainly on the dielectric medium between the conductors. It is the same as the velocity of unguided electromagnetic waves, in that dielectric medium. It depends also, but usually to a minor extent, upon the resistance and permeability of the conductors themselves, and hence upon frequency.

In hollow tube transmission the phase velocity of propagation depends upon the same quantities, but it depends also upon the diameter of the tube (in this factor differing from all other forms of propagation) and the type of wave being transmitted.

Fig. 3 illustrates the variation of the velocity of propagation and the wavelength of the waves within the tube with the exciting wavelength. It is drawn for a 10 cm. diameter metal tube filled with air and excited by an H_1-wave. The wavelength of the waves within the tube increases towards infinity as the exciting wavelength approaches the critical value. At exciting wavelengths much smaller than the critical one, the wavelength in the tube approaches the value of the exciting wavelength.

(*e*) *Attenuation Constant.* An important factor in transmission problems is that of attenuation or decrease of amplitude of the wave as it travels along the guiding system. The attenuation of waves along a co-axial cable increases as the square root of the frequency of the waves if dielectric losses are

FREQUENCY LIMITATION.

Type of Wave	Critical Wavelength	Critical Frequency
E_0	$1.31\ d\sqrt{k}$	$22{,}900/d\sqrt{k}$
E_1	$0.82\ d\sqrt{k}$	$36{,}600/d\sqrt{k}$
H_0	$0.82\ d\sqrt{k}$	$36{,}600/d\sqrt{k}$
H_1	$1.71\ d\sqrt{k}$	$17{,}500/d\sqrt{k}$

Critical frequencies and wavelengths for the four main waves in a cylindrical hollow metal tube (diameter d). If d is in cm., wavelength in cm., and frequency in Mc/sec., then for air, k = 1.

negligible. This is due to the corresponding increase in the resistance of the conductors with frequency.

The attenuation of waves along a hollow metal tube has been found by analysis. It varies according to the type of wave travelling along the tube. The variation of attenuation with the exciting wavelength

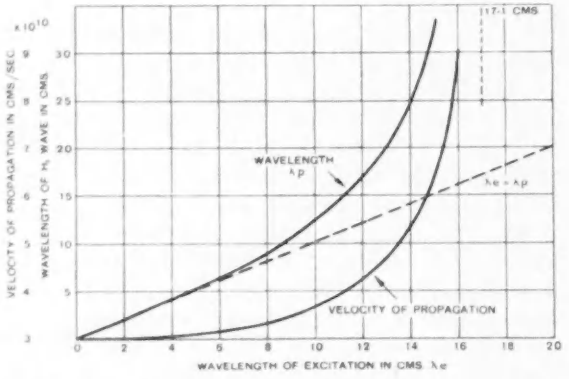


Fig. 3.—The velocity of propagation and the wavelength of an H_1 type of wave in a 10 cm. diameter hollow metal tube.

for the four main types of waves is shown in fig. 4.

The curves are drawn for a 10 cm. diameter copper tube filled with air. For all types of waves the attentuation approaches an infinite value at the corresponding critical wavelength. The attenuation then decreases rapidly with decreasing wavelength and for all types of waves, with the exception of the H_0-wave, the attenuation passes through a minimum value. For the E-waves, the minimum occurs at an exciting wavelength of 0·58 times the critical wavelength, and at 0·32 times the critical wavelength for the H_1-wave. For the latter wave there is a wide range of frequencies, about 4000 Mc/sec. where the attenuation is constant within ½ db. per mile. The attenuation characteristic of the H_0-wave is unique in that the attenuation decreases continually to zero with decreasing wavelength. However, analysis has shown that for even slight departures from the perfect circular cross-section of the tube, the attenuation characteristic assumes the general form and passes through a minimum value.

(*f*) *Characteristic Impedance.* In high frequency technique it is important to know the value of the characteristic impedance of a co-axial cable so that the transmitting and receiving terminal devices can be designed to obtain a maximum transfer of energy. The same is true of the hollow tube transmission. The characteristic impedance of a co-axial cable depends only upon the relative dimensions of the conductors and is practically constant at all frequencies.

In a hollow metal tube there is no return conductor and the usual concept of characteristic impedance, that of the impedance of an infinitely long tube, breaks down. The characteristic impedance can be defined by a second method in terms of the transmitted power. The results of such an analysis carried out on a 10 cm. hollow metal tube, filled with air, for the four main types of waves are given in fig. 5. At the critical wavelength for any wave, the impedance is zero and increases with decreasing wavelength.

(*g*) *Frequency Selectivity.* If a co-axial cable is of a given length and correctly terminated, it can be used as a resonant element. For example a length of cable, one quarter of a wavelength, shorted at one end acts as the equivalent of a parallel tuned circuit. In a similar manner the hollow metal tube cut to the correct length and correctly terminated can be arranged to be equivalent to a resonant element or to a circuit element of positive or negative reactance. For example a hollow metal tube of length equal to one wavelength of the waves within the tube and shorted at both ends will act as a resonant circuit.

The hollow metal tube is also in effect a high-pass filter and so offers a second frequency discrimination, allowing only frequencies above a certain value to be transmitted.

(*h*) *Excitation.* From a practical standpoint the method of excitation, to obtain the various types of waves in the hollow metal tubes, is of great importance. The transmitting equipment will consist of a two-conductor system. The electromagnetic energy

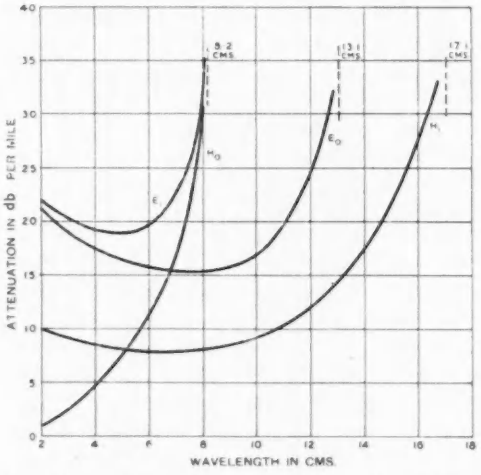


Fig. 4.—Attenuation in db. per mile for a 10 cm. diameter hollow copper tube for the four main types of waves.

is best transferred from this system to the hollow metal tube by means of co-axial cables. The outers are connected to the metal tube and the inners continued through the wall of the tube and placed along a line of electric force. Care is taken to ensure the correct phasing of the currents in the cables when two or more are used.

The method of excitation for the four main types

of waves is illustrated in fig. 6. For the E_0– and H_1–waves, a single exciting rod is used, but for the H_0– and E_1–waves, two rods are used, their currents being 180° out of phase with each other. In all cases the end of the hollow metal tube is closed by a movable piston. The position of this piston is varied to obtain a maximum transfer of energy from the exciting rods to the hollow metal tube.

The electromagnetic energy will be received at the far end of the tube in an exactly similar manner. The energy is re-transferred to a two-conductor system.

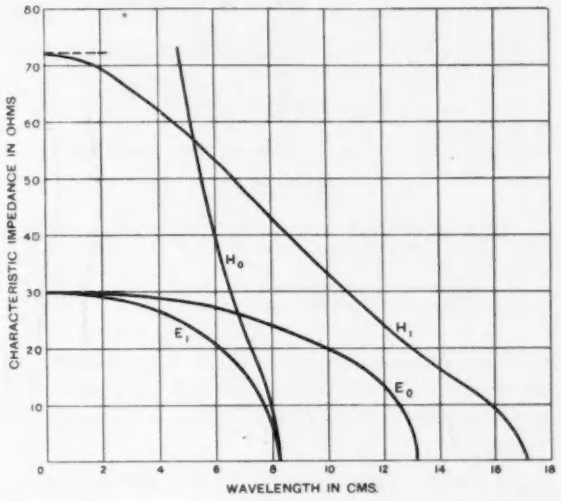


Fig. 5.—Characteristic impedance of a 10 cm. diameter hollow metal tube for the four main types of waves.

(*i*) *Brief Resumé of Experimental Procedure.* A large amount of experimental work has been carried out to verify the conclusions of the analysis. Using a small probe connected to a crystal detector and microammeter, it is possible to plot the approximate orientation of the electric field within the tube. A more elaborate detector, travelling along a slot in the hollow metal tube, can be used to measure the wavelengths of the waves within the tube. The wavelengths are found to be within 1 per cent of the calculated values. Attempts to measure the attenuation characteristics have shown that the form and magnitude of the calculated curves are correct.

Resonant chambers have been built and utilised. A valve and its associated circuits can be built within such a chamber, using it as the resonant circuit. An iris opening at one end of the chamber allows the egress of energy.

Very little work has been carried out upon the tubes as transmission lines. What has been done has shown that the energy can be readily transmitted around bends and sharp corners and that bad joints between sections of the tube have a negligible effect upon the transmission.

(*j*) *Practical Applications.* Before discussing the practical application of hollow metal tubes as transmission lines, it must be stressed that their use will be at wavelengths below 20 cm., as the absolute maximum diameter of tube allowable practically will be of the order of 10 cm.

It can be said in all fairness that of all forms of cross-sections, the circular one is the easiest and cheapest that can be produced. In analysis it is found to be in all points the equal of any other type of cross-section of tube. Using a cylindrical hollow metal tube of diameter 10 cm. it is possible to transmit the following ranges of wavelengths:

E_0-wave 0 to 13·1 cm.
E_1-wave 0 to 8·2 cm.
H_0-wave 0 to 8·2 cm.
H_1-wave 0 to 17·1 cm.

It will be seen that an H_1–wave can be transmitted along the tube at a higher exciting wavelength than any of the other types of waves.

The question of attenuation is, of course, the most important one from a communications standpoint. Returning to fig. 4, it will be seen that the *E*–waves can be ignored as their attenuation is greater, in all cases, than that of the *H*–waves. For a 10 cm. diameter tube from wavelengths 5 cm. to 17·1 cm., the H_1–wave has the least value of attenuation. Below 5 cm. wavelength the H_0–wave has a lower attenuation than the H_1–wave. But at 5 cm. wavelength a much smaller tube could be used to transmit the energy using an H_1–wave, giving a big saving in initial expenditure.

Following this discussion, it can be suggested with reasonable accuracy that for transmission line purposes, the H_1–wave propagated along a cylindrical hollow metal tube will find first application.

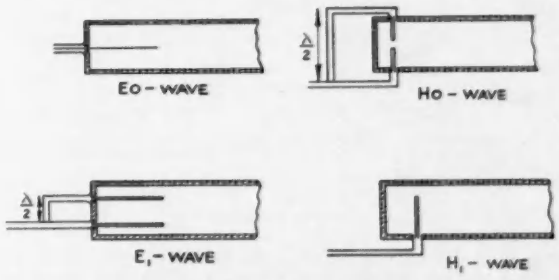


Fig. 6.—Method of excitation of cylindrical hollow tube for the four main types of waves.

The usual method of guiding electromagnetic energy is by means of co-axial cables. How does the hollow metal tube compare with the co-axial cable at similar wavelengths? In fig. 7 are drawn the attenuation curves for a 10 cm. diameter hollow copper tube, and for three diameters of co-axial cable, each of optimum design. The curves show that the co-axial cable has to be of the same order of

external diameter as the hollow metal tube to compare favourably with it.

If the hollow metal tube were to be filled with a dielectric medium, cheap, of low loss and high dielectric constant, its size for any given exciting wavelength could be reduced considerably. But it has been proved analytically that with the present standard of insulating materials, the losses along the tube would be far too great for the method to be of commercial application. The air filled hollow metal tube is then the only type which need be considered. But in a co-axial cable it is essential to use some form of insulating material between the inner and outer conductors. The losses in this material will further increase the attenuation above the values given in fig. 7.

Thus certain conclusions can now be drawn :—

(1) At wavelengths of 10 cm. and below, the cylindrical hollow metal tube, air filled and excited by an H_1–wave has an attentuation smaller than that of a co-axial cable, of zero dielectric losses and the same external diameter.

(2) Owing to the omission of the central conductor and the dielectric supports, the hollow metal tube would be simpler and cheaper to construct.

(3) Below 10 cm. the hollow metal tube has no commercial equal for the transmission of electromagnetic energy where a low attenuation is a necessity.

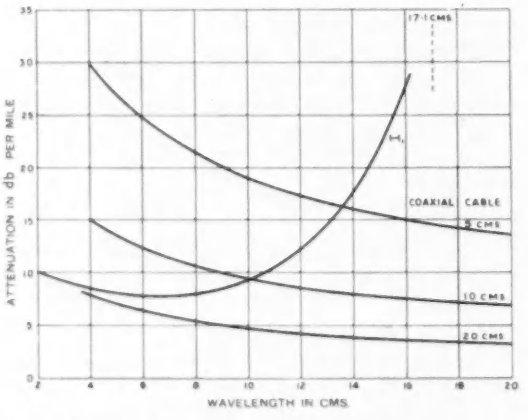


Fig. 7.—Attenuation in db./mile for a 10 cm. diameter hollow copper tube excited by an H_1 wave, and for three coaxial cables, neglecting dielectric loss. The coaxial cables are designed for minimum attenuation.

3. HOLLOW METAL TUBES AS RADIATORS OR RECEIVERS OF ELECTROMAGNETIC ENERGY.

(*a*) *Nature of Problem.* If an open-ended hollow metal tube be excited by electromagnetic waves, the latter will travel towards its open end. On reaching this end, a portion of the energy will be radiated into space and the remainder be reflected back along the tube. The efficiency of the transmission along the tube and the radiation into space is high. The radiant energy is propagated predominantly in the forward direction. If space waves of a frequency

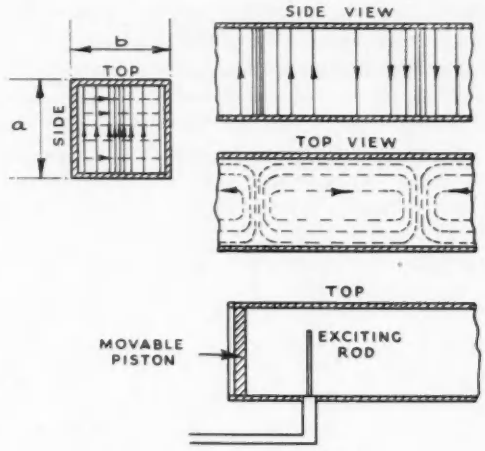


Fig. 8.—Approximate configuration of lines of electric and magnetic intensity in a rectangular hollow tube excited by an $H_{0,1}$ wave.
Lines of electric force ——>——
Lines of magnetic force - - - > - - -

above the critical frequency of the tube impinge on the open end of the tube, part will be transmitted along the tube to the terminal device where they will be received.

(*b*) *Hollow Tube Radiators.* For the discussion of hollow metal tubes as radiators a rectangular cross-section of the tube will be assumed. By analysis many types of waves can be propagated along the tube. Only one, namely the $H_{0,1}$–wave, will be discussed. The field orientation of this wave is shown in fig. 8. There is no component of electric field along the axis of propagation, and across any cross-section the lines of electric force are parallel and of half-sinusoidal distribution. This is the simplest configuration of any hollow tube wave. The critical wavelength is twice the width, *b*, of the tube, and is independent of the height, *a*.

It may be assumed that the field orientation of the $H_{0,1}$–wave produces, across the end of the tube, a displacement current sheet which has the current elements everywhere parallel to each other. Since the polarisation of the component space waves radiated by each current element is the same, the radiation will take place with maximum effectiveness. This feature is unique to the $H_{0,1}$–wave in a rectangular hollow tube and makes it especially applicable as a radiator.

The method of exciting the $H_{0,1}$–waves in the hollow tube is shown in fig. 8. The exciting rod is placed along a line of electric intensity. A movable piston is used to obtain the maximum transfer of energy along the tube. The only adjustment on

changing the frequency of excitation is the position of the piston relative to the exciting rod.

The radiation pattern, comprising the curve of the amplitude of the electrical intensity in space plotted against the space angle, is the most important feature of any radiator. This has been found for the hollow tube radiator by analysis. The theory is of necessity approximate because of the impossibility of satisfying the boundary conditions at the open end of the tube. It is sufficiently accurate for practical purposes.

For convenience it is usual to plot only two radiation patterns. One is in the vertical plane, which is parallel to the side, *a*, of the tube. The other is in the plane parallel to the width or side *b*,

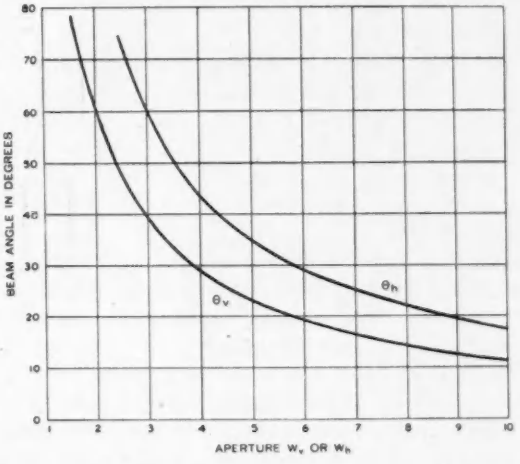


Fig. 9.—The horizontal (θ_h) and vertical (θ_v) beam angles for a rectangular hollow tube radiator excited by $H_{0,1}$ waves.

and is in the horizontal plane. The angle subtended at the horn by the principal lobe of the radiation pattern is known as the vertical beam angle, θ_v. The corresponding angle in the horizontal is known as θ_h. In the determination of these angles a special designation has been made. The vertical aperture is denoted as the length of the side, *a*, in terms of exciting wavelengths, and is called W_v. The corresponding horizontal aperture is termed W_h. By analysis it has been found possible to express θ_v and θ_h in terms of W_h and W_v. This is shown in fig. 9. For equal apertures it is found that the beam will be sharper in the vertical plane than in the horizontal plane by a ratio of 2 : 3. If equal beam angles are to be obtained in the vertical and horizontal planes the ratio of width, *b*, to height, *a*, is equal to 3 : 2. By changing the relative magnitudes of *b* and *a* it is possible to obtain a beam having very different angles in each plane.

(*c*) *Horn Radiators.* This type of construction is shown in fig. 10. For convenience it has been shown as a horn flared in the horizontal direction only, the flare angle being φ. It can, of course, be flared in both directions. The horizontal aperture of the horn is much larger than the vertical aperture. The methods of exciting the horn are numerous. The exciting rod can be placed immediately in the throat of the horn, or the radiator may be fed by means of a hollow rectangular metal tube of cross-section equal to that of the throat and excited in the normal way.

Only one type of wave propagated along the horn will be discussed. This is the $H_{0,1}$–wave. The field orientation within the horn is shown in fig. 10. It will be seen that the electric field is composed of lines parallel to the exciter across the whole cross-section. Thus the radiated space component has all its elements polarized exactly the same and the radiation is at its maximum efficiency. If a hollow metal tube is used to feed the horn, then the tube will itself be excited by its own $H_{0,1}$–waves. At the throat these waves will be converted to the horn type of $H_{0,1}$–wave and propagated along the horn and so into space. The wavelength of the waves within the horn varies along the length of the horn, increasing as the waves approach the open end. In order that waves can be efficiently propagated along the horn its throat must be situated a certain distance from the hypothetical apex. If the throat is within this distance, the horn will only propagate a small portion of the waves and the remainder will

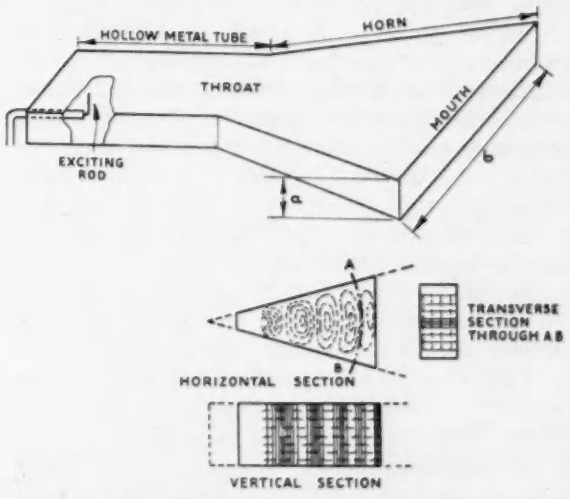


Fig. 10.—Construction and excitation by $H_{0,1}$ waves, of a rectangular section flared horn, and the approximate configuration of lines of electric and magnetic force.
Lines of electric force ——>——
Lines of magnetic force - - - > - - -

be stored as electrical and magnetic energy. This, of course, corresponds to the critical diameter of a cylindrical hollow metal tube, but in the case of the horn waves this critical value is not so well defined.

The radiation characteristics of the horn have been found by analysis. The most important result of this analysis is that for a given length of horn there exists an angle of the flare to give a minimum horizontal beam angle. For lengths of horn between 8 and 10 wavelengths this optimum flare angle is

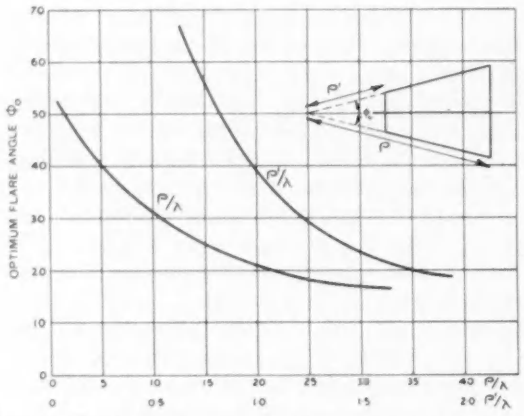


Fig. 11.—Optimum flare angle for minimum beam angle for a horn flared in the horizontal direction and excited by $H_{0,1}$ waves (ρ/λ curve).
Minimum distance of mouth of horn from apex for efficient transmission of $H_{0,1}$ waves at this optimum flare angle (ρ'/λ curve).

about 32°. A second important result of the analysis is that to a first approximation for flare angles up to the optimum the radiation pattern is the same as that of a rectangular hollow tube of dimensions equal to the open end of the horn. Thus the radiation pattern for the horn can be calculated simply from the theory of the hollow tube radiator. The horn radiator is, then, a form of hollow tube radiator of much smaller overall dimensions but possessing the same radiation characteristics if the open end apertures are the same as those of the hollow tube.

In fig. 11 is shown the relation between the optimum flare angle to give minimum beam angle and the length of horn in wavelengths; also the relation between this flare angle and the distance that the throat of the horn should be from the apex for efficient propagation. In combination with fig. 9 there are now complete data to design a horn radiator of optimum flare angle and giving the required horizontal or vertical beam angle.

(*d*) *Radiation Patterns.* A number of calculated radiation patterns for rectangular hollow tube radiators and for horn radiators are shown in fig. 12.

In the first set is shown the shape of the horizontal radiation pattern obtainable with a hollow metal tube of width 50 cm. and length about 5 m. The values of horizontal aperture, W_h, are not large for any of the four patterns. There is a definite narrowing of the beam at an exciting wavelength of 52 cm. compared with that at 98 cm. The latter wavelength is within 2 per cent of the critical wavelength of the tube.

The second set of patterns consists of the horizontal radiation characteristics for a horn of constant length 1·5 m. and of throat width 50 cm. excited by a wavelength of 50 cm. Each pattern is drawn for a certain flare angle of the horn and so for different horizontal apertures. They have been drawn on the assumption that the horn behaves as a rectangular hollow tube of dimensions equal to the mouth of the horn. This assumption is correct for flare angles up to 40°, but the pattern for a flare angle of 60° is narrower than would be obtained in practice. In fact, due to the existence of an optimum flare angle, the pattern at a flare angle of 60° would actually be wider than that at 50°. The patterns show remarkably well the effect of increasing aperture upon the width of the beam.

(*e*) *Brief Resumé of Experimental Results.* The experimental work carried out on hollow tubes and horn radiators has been, mainly, to verify the analysis. The measured horizontal radiation patterns for a hollow metal tube have been shown to agree with the calculated patterns to such a degree that the theory can be used for any required practical design. The radiation pattern can only assume the smooth form predicted if the length of the tube is greater than 8 wavelengths. For shorter lengths of tube the secondary lobes become enlarged and destroy the smoothness of the pattern. The presence of these lobes is due to the departure from a sine wave distribution of the electric intensity across the mouth of the tube. This departure is due to the length of the tube not being great enough to allow the proper formation of hollow tube waves.

For electromagnetic horns it has been shown that an optimum flare angle does exist to give a minimum beam angle at any given wavelength and length of horn. Measured horizontal patterns for the horn have shown that for flare angles up to the optimum the assumption that the pattern is similar to that for a hollow tube of the same dimensions as the mouth of the horn is correct. The length of the horn must be at least 6 wavelengths for the predicted radiation pattern to be obtained. There is very little back radiation or secondary lobes if this condition is satisfied.

(*f*) *Practical Applications.* It has already been shown that below 10 cm. wavelength the hollow metal tube has no equal for the transmission of electromagnetic energy. The wavelength limitation is not so harsh upon the hollow metal tube when used as a radiator. Initial expenditure is not the most important factor in the building of a suitable radiator. For a beam angle of 40° the horizontal aperture has to be of the order of 5. At a wavelength of 50 cm. this means the mouth of a horn has to be 250 cm.,

its length approximately 300 cm. with the exciting rod placed near the throat of the horn. These dimensions are not too great to make the horn a practical suggestion for beam propagation at 50 cm. wavelength, especially as its construction is simple.

What, then, are the advantages of using such a scheme compared with other systems of radiators? The secondary lobes are of small magnitude, and there is very little back radiation. Compared with the parabolic reflector or broadside array systems of antennae of similar bulk, the horn radiator is superior in this respect. To give maximum radiation at any required wavelength the only adjustment is the position of the exciting rod relative to the throat of the horn, and this adjustment itself is not critical. The cylindrical parabolic antenna has a critical adjustment of focal distance. In the case of the broadside array, which has a slight advantage in bulk over the horn radiator for a given beam angle, the adjustments are critical for any given wavelength and difficult to make.

The radiation pattern of the horn is calculated simply and can be predicted accurately. It is not critically dependent on wavelength. The other forms of antenna contain resonant elements and not only is the radiation pattern difficult to predict but it alters rapidly with wavelength.

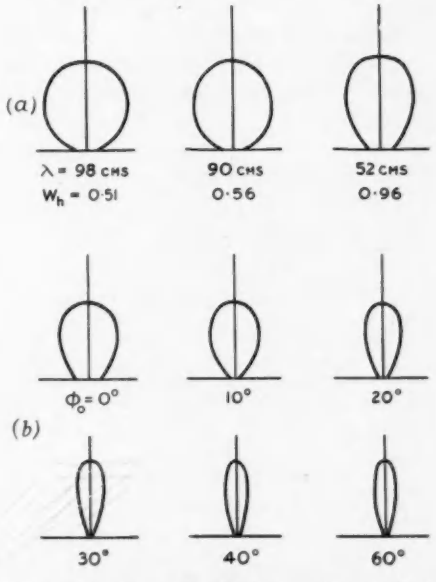


Fig. 12 (a)—Horizontal radiation patterns (calculated) for a rectangular hollow tube radiator (b = 50 cm.) excited by $H_{0,1}$ waves.

(b)—Horizontal radiation patterns (calculated) for a flared horn radiator, rectangular cross-section, of constant length and excited by $H_{0,1}$ waves of a frequency of 600 Mc/sec. (50 cm.)

One of the main difficulties in obtaining a predicted radiation pattern with conventional arrays is to obviate spurious radiation from the insulating supports and the feeder line. With a horn there is no necessity for insulating supports at the mouth of the horn, and the feeder line is brought into the horn at the remote end. Spurious radiation is then reduced to a minimum. Thus for beam propagation where a smooth, predictable pattern is required at wavelengths below 50 cm., the electromagnetic horn has no equal among conventional directive radiators.

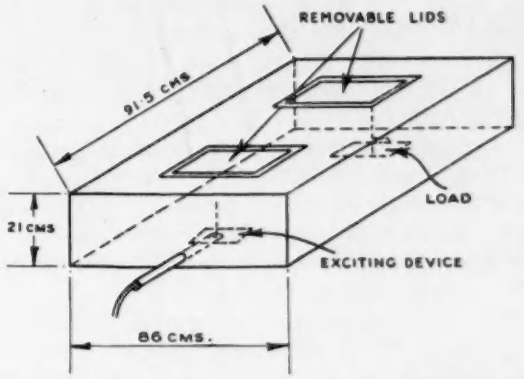


Fig. 13.—Constructional details of resonant chamber and the methods of exciting it with $H_{0,1}$ waves, and of loading the chamber.

4. ORIGINAL EXPERIMENTAL WORK.

(*a*) *A Resonant Chamber.* A practically uniform electric field over an area of 20 cm. square was required for high frequency experimental work at a wavelength of 80 cm. In order to obtain this, a resonant chamber was built.

It was decided that the rectangular cross-section hollow tube excited by $H_{0,1}$-waves was the most practical form. To provide efficient transmission of energy with little loss of power the width of the tube was made 86 cm. To enable the use of a $\frac{1}{2}$-wavelength exciting rod, the tube was made 21 cm. high. For a tube of these dimensions excited by an $H_{0,1}$-wave the wavelength of the waves within the tube would be 91·5 cm. Thus a box of cross-section 86 cm. × 21 cm. and length 91·5 cm. closed at both ends, formed the required resonant chamber. This is illustrated in fig. 13. The box was built of aluminium. The movable exciting rod was positioned at approximately $\frac{1}{4}$-wavelength from the end of the box. The area of uniform field was at the corresponding position at the opposite end of the box.

The resonant wavelength of the box was found to be 82 cm. The field was found to be reasonably uniform over the required area. The efficiency of transmission from the oscillator to the load was 80 per cent*. The practical adjustments were the positioning of the exciting rod and the wavelength of excitation, neither of which was found to be unduly critical.

* This includes losses in the feeders amounting to 15 per cent of the oscillator output.

(*b*) *An Electromagnetic Horn Radiator or Receiver.* A beam propagation for wavelengths between 10 and 15 cm., sharp in the horizontal plane only was required. A horn flared in the horizontal plane and excited by $H_{0,1}$–waves was considered to be the best form for the radiator.

It was decided to build the horn for a horizontal beam angle of 40° at 10 cm. wavelength and the angle of flare to be the optimum at this wavelength. To ensure the correct configuration of the $H_{0,1}$ –waves at the mouth of the horn, the latter has to be 8 wavelengths (80 cm). from the apex. From fig. 11 the optimum flare angle for this length of horn is found to be 32°.

The horn must efficiently radiate wavelengths of 15 cm., so that from fig. 11 it will be seen that, for a flare angle of 32° the throat of the horn must be 18 cm. from the apex. The height of the horn is immaterial and is fixed at 5 cm. to allow the use of a $\frac{1}{4}$-wavelength exciting rod.

The dimensions of the horn are now fixed and are given in fig. 14. It was decided to feed the horn by means of a length of hollow metal tube of width equal to that at the throat of the horn. This allows

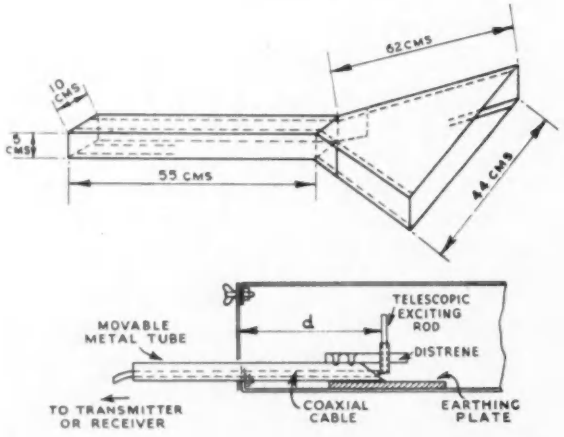


Fig. 14.—(Above), details of horn; (below), details of method of excitation.

for a maximum wavelength of 20 cm. to be transmitted, though not efficiently. The length of the hollow tube is approximately 5 wavelengths. The horn was constructed of tin-plate. The method of excitation is shown in fig. 14. The position of the exciting rod relative to the closed end of the tube can be varied. A magnetron oscillator was used to generate the power at the required wavelengths. The exciting rod was fed by means of a co-axial cable.

In order to find the radiation pattern of the horn experimentally a second and similar horn was built. This was not essential but a receiver responsive to a vertically polarized field had to be used in order to

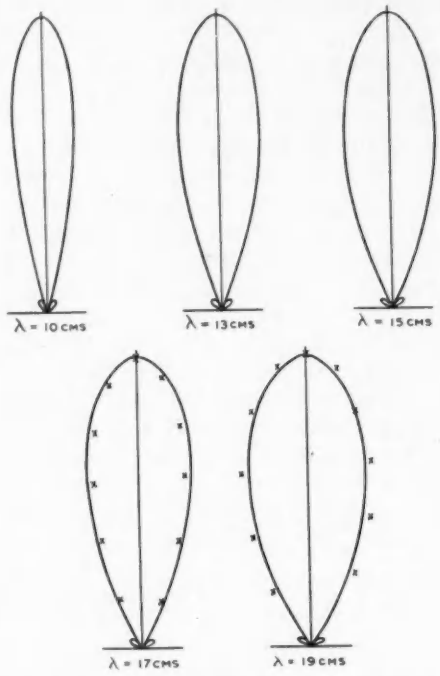


Fig. 15.—Radiation patterns. The points indicated by *x* represent experimental measurements.

minimise the effect of room reflections. For the receiving horn the exciting rod became the receiver and was connected to a thermocouple and galvanometer to measure the electric intensity at the exciting rod. The transmitting horn was kept fixed in position. Keeping the centre of the mouth of the receiving horn fixed in position along the axis of the transmitting horn, the closed end was rotated. The radiation pattern of the receiving horn, and hence the transmitting horn was plotted.

In fig. 15 are shown the predicted radiation patterns of the horn at wavelengths of 10, 13, 15, 17, 19 cm. and the actual measured patterns at wavelengths 17 and 19 cm.

Using the two horns it is thus possible to obtain a very efficient beam link between a receiver and a transmitter.

CONCLUSION

The paper has been intended as a survey of the uses and characteristics of hollow metal tubes for the propagation of electromagnetic energy. A brief summing up will indicate that :—

(1) At wavelengths below 10 cm., the cylindrical hollow metal tube, excited by H_1–waves, will find many applications where a high efficiency of transmission of energy is required ; and

(2) At wavelengths below 50 cm., a hollow metal tube, probably of the horn type, will find application where a narrow predictable beam propagation is needed.

In support of this latter statement, the results of tests in America can be reported. At East Boston Airport, by the use of two horn radiators, at 40 cm. wavelength, the blind landing of an airplane has been successfully carried out. These results show that there is a most definite immediate future in the use of hollow metal tubes as radiators at these wavelengths.

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The plant at this quarry for washing and screening sand and gravel was supplied by the Fraser & Chalmers Engineering Works of The General Electric Co., Ltd. The illustration shows the plant in operation and the material in process of being screened and conveyed to the various bins, whence it is discharged into lorries.

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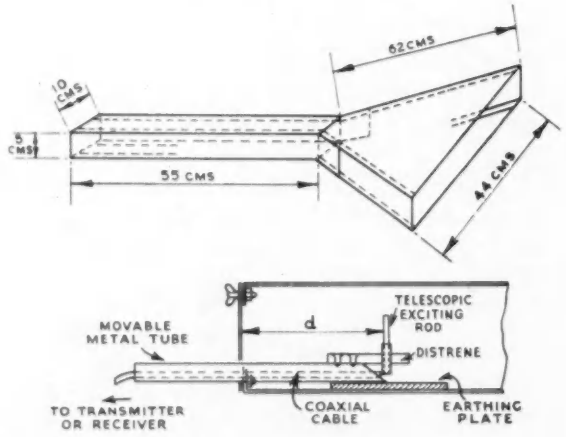

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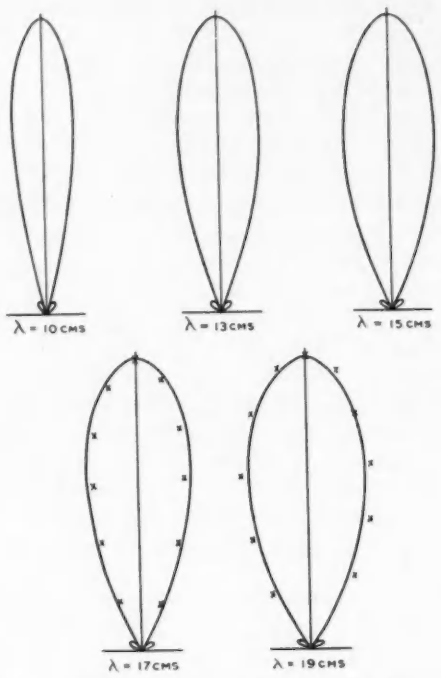

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A Precision Thermionic A.C. Test Set.

By K. A. MACFADYEN, M.Sc., and N. D. HILL, B.Sc., A.Inst.P.
G.E.C. Research Laboratories.

The current consumption of voltmeters and of the pressure coils of wattmeters is a disadvantage which may preclude their use for certain purposes. With a view to removing this restriction a combined A.C. voltmeter, ammeter and wattmeter has been made in experimental form, incorporating a specially stabilized thermionic amplifier interposed between the voltage terminals of the load and the voltmeter or pressure coil. Special attention has been paid to the accuracy and stability of the instrument, which can be used over the frequency-range of 50 to 1,000 cycles/second and hence is suitable for dealing with distorted waveforms. The equipment is at present in a semi-permanent form whilst undergoing a long-term laboratory trial in connection with power measurements on low wattage electric discharge lamps.

1. INTRODUCTION.

THE current consumption of voltmeters and of the pressure coils of wattmeters limits the sphere of usefulness of these instruments. Whilst it is right to say that such instruments indicate the true value of voltage or power[1] in the load at the moment of measurement, errors can arise in that the act of connecting up the meter may modify the operating conditions of the load.

A particular example of this trouble is presented by the case of the low wattage electric discharge lamp. Here the lamp itself, upon which it may be necessary to carry out measurements, is connected to the A.C. mains through a stabilizing choke or leakage-type transformer. The consumption of a voltmeter or wattmeter connected across the lamp not only reduces the voltage by virtue of the series reactance of the choke or transformer, but may easily modify the circuit conditions in such a way that the lamp "starts" at a different part of the A.C. cycle after the meter is connected. In extreme cases the lamp may fail to light at all with the meter in action.

An inherent property of electric discharge lamps operating on alternating current is that considerable distortion is unavoidably present in either the current or voltage waveform, or in both. This makes it impossible to apply any form of compensation for the type of error under consideration. The only solution is to use measuring instruments which do not draw an appreciable current from the circuit.

The procedure in regard to voltage measurements, if the voltage is high enough, is to use an electrostatic voltmeter, but the indications are that a wattmeter working on the electrostatic principle would need to be a very delicate instrument if it were to give the sensitivity and accuracy required.

Various forms of thermionic wattmeter have been devised, incorporating modulator circuits which produce a direct current whose value is proportional to the instantaneous product of two alternating voltages, derived respectively from the load voltage and load current. These devices possess negligible current consumption, but they are subject to variations in sensitivity arising from changes in valve characteristic and fluctuations in the voltages of the supplies to the anodes and heaters.

When the problem of power measurements on low wattage discharge lamps became acute, the foregoing schemes were considered. The first was abandoned because a sturdy and reasonably portable instrument was desired, whilst the second was also rejected because the class of work for which such an instrument would be used in the Research Laboratories demanded "sub-standard" accuracy.

In view of this somewhat stringent condition, it was decided to attempt the construction of an experimental instrument embodying as much of the existing technique of power measurement as possible.

2. PRINCIPLE OF THE INSTRUMENT.

Accordingly, it was proposed to employ a dynamometer wattmeter of high accuracy, the pressure coil of which would be fed through a thermionic amplifier. The input impedance of the latter would be so high that it could not materially affect the operation of even the smallest discharge lamp.

It might be argued that such an arrangement would be subject to the drawbacks of the thermionic wattmeter already described, but it is now possible, by utilizing the principle of negative feed-back, to construct amplifiers having extremely stable properties. Since the amplifier under consideration acts merely as a kind of "high-impedance buffer" between the lamp and the pressure coil, the application of negative feed-back becomes possible. The current coil is placed in series with the lamp in the usual way.

(1) It is assumed here that the wattmeter is fitted with an internal compensating coil.

The main problems which have to be solved before such a system becomes workable are two in number :—

1. The phase-shift of the amplifier must be very small. When the power factor of the load is low, the accuracy of a wattmeter rapidly deteriorates with phase error in the pressure coil circuit. It was decided to aim at a maximum error of $\frac{1}{2}°$ so as to bring the instrument into class "A." of the British Standards Institution.
2. The gain of the amplifier must remain constant within limits of $\pm\frac{1}{2}$ per cent against the usual fluctuations in mains voltage. This is also based on the requirements of the B.S.I. "Class A" specification.

It will be shown later how these conditions were satisfied, but at this point it would be as well to notice that the successful construction of such an amplifier opens up a considerable field for the use of thermionic valves in A.C. measurements. In any case where the current consumption of a voltage-operated instrument is a source of error or inconvenience, the use of a stable amplifier is a possible solution. It then becomes practicable to employ the most reliable and accurate type of A.C. voltmeter, namely the dynamometer, in many cases where the heavy power consumption of this type of meter would otherwise preclude its use. Furthermore, the use of the amplifier enables the range of a meter to be extended in both directions without loss of accuracy. As an example of this, the test set described here can be used as a voltmeter of practically infinite resistance, with ranges from 750 volts down to 30 volts. The lowest practicable range for a pivot-and-pointer type of electrostatic voltmeter is 120 volts.

Care must be taken not to confuse the present instrument with a "valve voltmeter," in which the final indication is due to the rectifying action of a valve. The present system must necessarily incorporate a true A.C. instrument, and the accuracy of measurement depends to a large extent on the instrument chosen for the purpose. If this is inherently accurate the accuracy attainable by this method appears to be high.

3. CIRCUIT OF STABILIZED AMPLIFIER.

It has been shown[2] that when a fraction of the output voltage of an amplifier is fed into the input terminals in addition to the ordinary input voltage, the gain of the amplifier is effectively increased or reduced according to the sense in which the feed-back connections are made. The gain is changed from its original value μ to a new value μ' where

$$\mu' = \frac{\mu}{1 - \mu\beta}$$

Here β is the fraction of the output voltage fed back to the input terminals. Assuming that the gain, μ, is positive, *i.e.*, that the output voltage is in phase with the input, the introduction of a phase reversal into the feed-back path will cause a reduction in gain. This condition is called "negative feed-back," since it involves a negative value of β due to the phase

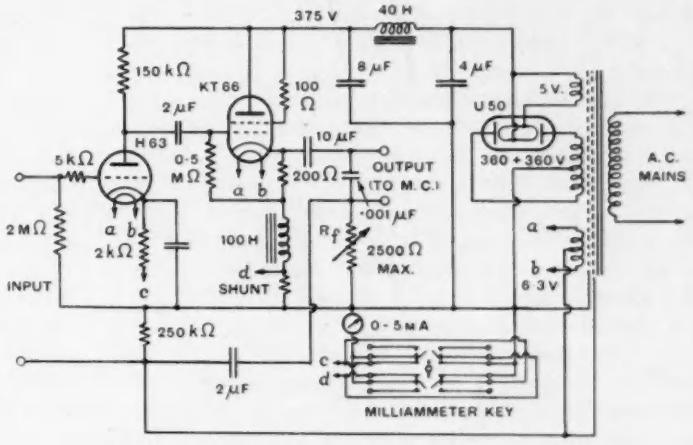


Fig. 1.—Circuit diagram of stabilized amplifier.

reversal. The condition may also be established by employing a direct feed-back path with an amplifier giving 180° phase-shift. In this case μ is negative instead of β, and the reduction in gain takes place as before.

The importance of this procedure lies in the fact that, simultaneously with the reduction in gain, there are proportionate reductions in phase-shift and in the distortion at a given output. When the original gain, μ, is high, and when β approaches unity it is possible for the figure 1 in the denominator of the above expression for μ' to be negligible in comparison with the product of $\mu\beta$. The modified gain is then equal to $\frac{1}{\beta}$, that is to say, it depends only on the properties of the feed-back circuit, not on the gain of the valves. Variations in gain due to mains fluctuations no longer affect the overall performance of the amplifier.

It will immediately be obvious that these properties are all that could be desired in an amplifier intended for incorporation in a precision instrument. It is only necessary now to decide what form the circuit shall take.

Negative feed-back circuits can be divided into three main classes, according to the way in which the feed-back voltage is derived. In the first class it is arranged to be a definite fraction of the output voltage. In the second it is proportional to the output

(2) E.g. By H. S. Black. *Bell System Technical Journal*, February, 1934.

current. The third system derives its feed-back voltage partly from the output voltage and partly from the current. The theory considered above is applicable to all three systems, but the distinction between them lies in their differing response to changes in terminating impedance. The first class of amplifier tends to maintain a constant voltage across the termination, whilst the second tends to keep a constant output current, in spite of variations in impedance. In short, these two types of circuit behave respectively as though they had very low and very high (internal) output impedances. In the third system the impedance can be adjusted to have any desired value between the limits set by the other two systems.

The present problem is to pass through a pressure coil an alternating current whose value, phase and waveform accurately conform with the input voltage, which is derived from the lamp or other apparatus on test. Since the coil is unavoidably inductive, its impedance increases with frequency. Hence, to avoid phase and ratio errors at the higher frequencies the second class of circuit, the so-called "current feed-back" system, is chosen.

A considerable amount of work was expended with the object of getting the desired results with only one stage of amplification, but it eventually appeared that the required degree of stability and freedom from error necessitated two stages. It might be supposed that this would give a higher final gain, but a little consideration of the formula in Section 3 will show that the object of increasing the intrinsic amplification was to increase the product $\mu\beta$, making the final gain more independent of μ than before. In brief, the extra stage was added so that the amplification accruing from it could be "thrown away" in the form of extra feed-back to secure stability and fidelity.

The main features of the circuit finally selected are shown in fig. 1. The milliammeter key is merely part of the monitoring arrangements and need not be considered at the moment. A valve, type H63, of high amplification is resistance-capacity coupled to a triode-connected KT66, a power valve capable of passing a standing current of 80 milliamperes. The load of this valve is in its cathode circuit, an arrangement designed to maintain the phase of the feed-back voltage in the correct relation. It has the further advantage that the condenser which passes the alternating output current into the pressure coil (through the output terminals) need not be capable of sustaining a high D.C. potential. The current in the pressure coil is forced to traverse a resistance marked R_f in fig. 1, where it builds up the feed-back voltage. In passing, it may be mentioned that the usual resistance connected in series with the pressure coil of the wattmeter is not retained when an amplifier is used, since, far from serving a useful purpose, it makes the design of the amplifier much more difficult.

The voltage developed across the "feed-back resistance" R_f is transmitted back to the input circuit by way of a network consisting of a 2 microfarad series condenser and a $\frac{1}{4}$ megohm shunt resistance. The object of this is to correct the residual phase-shift at 50 cycles/second due to the coupling components in the amplifier. The residual shift is only about $\frac{1}{2}°$ and diminishes at higher frequencies, but this simple device reduces it to a few minutes of arc at 50 cycles/second.

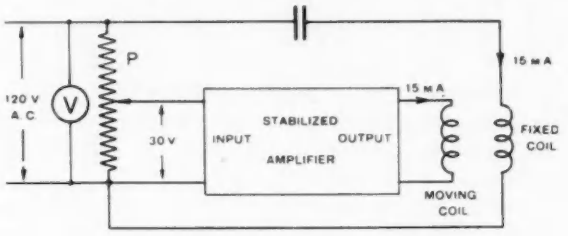


Fig. 2.—Circuit for phase-error test.

The resistance R_f serves an important purpose as well as that of producing the feed-back voltage, for by adjusting R_f, the gain of the amplifier may be varied over a narrow range. This provision is necessary in order to set the instrument to give correct readings. Once this has been done, further adjustments to R_f should not be necessary.

4. PERFORMANCE OF THE AMPLIFIER.

The dynamometer chosen for use with the stabilized amplifier is of the double-pivot "high-efficiency" type. The special feature of this kind of dynamometer is that, by the use of laminated cores of high permeability alloy, a much improved utilization of the feeble magnetic field of the fixed and moving coils is brought about. The practical result is that, for a given deflecting torque, the power consumption of the instrument is greatly reduced as compared with the consumption of an air-cored dynamometer. This development in instrument design is due to the Cambridge Scientific Instrument Company, working on suggestions originally made by Dr. Drysdale.

The model in question is designed to give a full-scale deflection with 15 milliamperes in both coils, which are of comparable impedance. This current is delivered by the amplifier when an input voltage of 30 volts R.M.S. is provided.

In addition to the problem of constructing an amplifier having the required properties, the writers were faced with the necessity of measuring the phase-shift of the complete unit and verifying that the gain was independent of frequency. This was done by means of the circuit shown in fig. 2. In this

arrangement a measured voltage across the potentiometer P simultaneously provided an input to the amplifier and a known current in quadrature through the fixed coil. If the amplifier had given no phase-shift there would have been no deflection of the dynamometer. The deflection actually produced was a measure of the phase-shift. A correction for the power factor of the fixed-coil circuit was applied, but as the impedance of the condenser was very great compared with that of the coil the correction was small.

The phase-correcting circuit mentioned in the last section was adjusted to compensate for the $\frac{1}{2}^\circ$ of phase error measured in this way at 50 cycles/second. At 100 cycles/second the error was also compensated but at 400 cycles/second $\frac{1}{2}^\circ$ was indicated, whilst at 1,000 cycles/second the error had risen to $1\frac{1}{2}^\circ$. This is probably well within the limits of the most accurate wattmeters of the ordinary type.

An investigation of the dependence of gain upon frequency presented no particular difficulty. The method adopted was to connect the fixed and moving coils in series in the output circuit of the amplifier and to note the indication at various frequencies when a standard voltage, as tested by a sensitive electrostatic voltmeter, was applied to the input terminals. The reading was found to be constant within ± 0.1 per cent over the frequency-range 50–1,000 cycles/second. When the mains voltage was changed from 200 to 260 volts the change in calibration was found to be less than 0·1 per cent. Thus, with the ordinary fluctuations in supply voltage the error is never likely to be appreciable.

5. USE OF THE AMPLIFIER IN AN A.C. TEST SET.

The practical embodiment of the principle of using the stabilized amplifier in conjunction with an accurate dynamometer is illustrated in fig. 3. In this test set the amplifier is provided with a capacity input-potentiometer so as to provide 5 voltage ranges. The current side is provided with 7 ranges by means of a current transformer of toroidal form. Used as a wattmeter, its range is from 0·4 watt to 1 kilowatt. Two wattage scales are provided so as to permit any of the 35 possible combinations of the current and voltage ranges to be used without mental calculations or reference to curves. It is only necessary to select the appropriate ranges by means of two radial stud switches and to place the central selector switch in the position "W."

To convert the instrument to a multi-range, high impedance voltmeter the selector is turned to the position "V." For use as an ammeter it is placed at "A." All of the five voltage and seven current ranges are direct reading from a third scale on the instrument.

The circuit of the switching system is shown in fig. 4. The main item of interest in this is the capacity potentiometer which provides the multi-range feature on the voltage side. This is dealt with in the following section.

The press-buttons P_1 and P_2 are provided to prevent accidental overloads, although it may be mentioned in this connection that the amplifier acts as a partial safeguard, since it is incapable of delivering more than about 70 milliamperes R.M.S. into the meter.

It will be gathered from fig. 4 that there is no need to disengage the voltage terminals when using the instrument as an ammeter, or vice versa, provided that only the appropriate push-button is depressed. There is no conducting path between the two pairs of terminals.

The two 3-stud switches shown in fig. 4 are ganged to form the selector switch in the centre of the panel, shown in fig. 3.

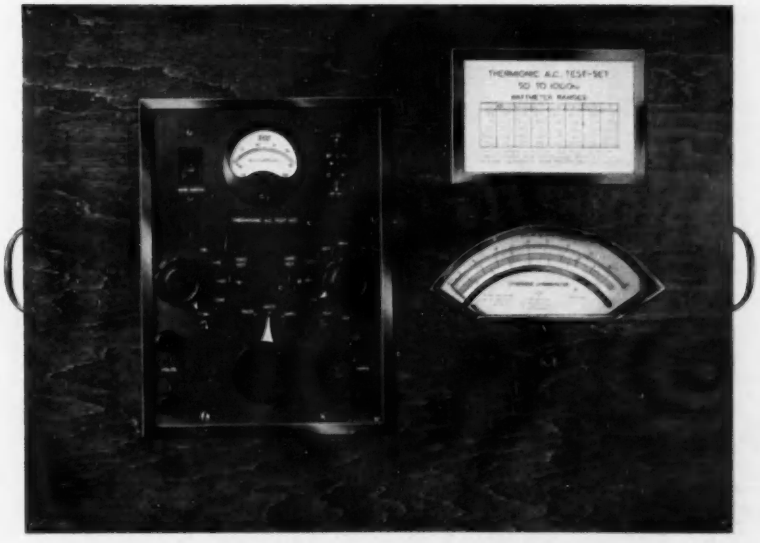


Fig. 3.—A.C. thermionic test set.

The full selection of current and voltage ranges is shown in Table 1.

6. THE CAPACITY POTENTIOMETER.

One useful property of the feed-back amplifier not already mentioned in Section 3 is the fact that the application of negative feed-back increases the input impedance of an amplifier by the factor $1-\mu\beta$. In the present case this factor is about 80, with the result that the input impedance is about 160 megohms. It would have been possible to employ a resistance potentiometer in order to extend the range, but the extremely low current consumption required for certain applications of the instrument would have necessitated a potentiometer of at least 75 megohms. Apart from the practical difficulty of making such a potentiometer, there would have been serious errors due to stray capacities.

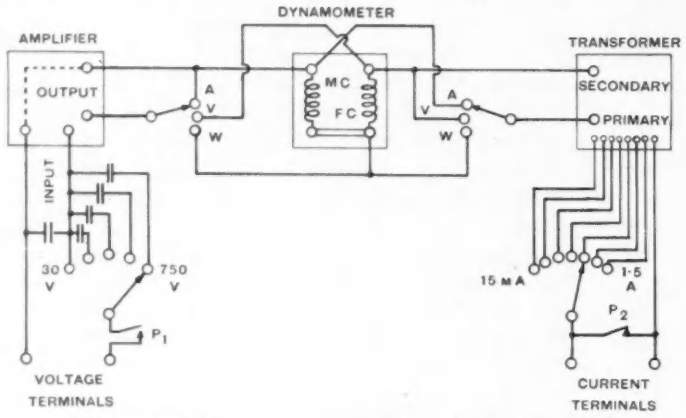


Fig. 4.—Switching arrangements in multi-range test set.

The difficulty was solved by using a capacity potentiometer. Ceramic condensers of great stability are used in its construction. By keeping the wiring short and fixing it firmly in position the necessary constancy of properties is secured without unduly decreasing the high impedance at the input terminals. On the 30 volts range this is 300,000 ohms (capacitive) at 50 cycles per second, rising to about 7·5 megohms for the 750 volts range. At higher frequencies, of course, the impedance is proportionately lower.

7. MISCELLANEOUS ERRORS AND THEIR PREVENTION.

Little need be said about the current side of the apparatus because the instrument transformer was designed on conventional lines. A ring-core made of laminated Mumetal was employed, and the windings were disposed upon it in such a way that no matter which current range was in use, the operative portion of the primary extended around the entire ring. These precautions resulted in phase and ratio errors low enough to admit the transformer to the B.S.I. Class A category.

In any amplifier fed from rectified A.C. there is always a slight admixture of 50 or 100 cycle current in the output circuit. This is usually called hum. In the present case, fortunately, hum due to the valves themselves is negligible, but trouble was at first experienced from the passage of stray currents from the power unit through the feed-back resistance. The effect of these was visible as a slow oscillation of the pointer when the generator feeding the measurement side of the apparatus was giving a slightly different frequency from the mains supplying the power unit of the amplifier. The trouble was eliminated by using a double-screened mains transformer with the screens connected to appropriate points in the circuit, as illustrated in fig. 1.

When dealing with extremely distorted wave-forms it is necessary to ensure that the measuring apparatus is capable of handling peaks of current or voltage many times higher than the R.M.S. value.

TABLE 1.

WATTMETER RANGES.

Volts	Amps.						
	0.015	0.030	0.075	0.150	0.300	0.750	1.5
30	0.4	0.8	2.0	4.0	8.0	20	40
75	1.0	2.0	5.0	10	20	50	100
150	2.0	4.0	10	20	40	100	200
300	4.0	8.0	20	40	80	200	400
750	10	20	50	100	200	500	1000

For Ranges 4, 8, etc., use Scale 0—8.
For Ranges 2, 5, 10, etc., use Scale 0—10.

This condition makes it necessary, in the present case, to design the amplifier with a wide margin of safety. It is, in fact, capable of delivering instantaneous peaks of three times the amplitude of the sinusoidal current required to give full-scale deflection.

It is important to remember that like any meter incorporating an instrument transformer the present instrument ignores D.C. components of current and voltage. This, however, is no drawback for the majority of applications.

8. MONITORING EQUIPMENT.

Many technicians who would otherwise employ thermionic measuring instruments regard them with mistrust because of the finite life-period of the thermionic valve. It is argued that since the valves ultimately fail there must be a period during which the accuracy of the instrument deteriorates. Such a prospect would make frequent calibration essential.

It has already been shown that the use of the negative feed-back principle renders the performance almost independent of variations in valve characteristics. If a check is kept on the anode currents of

the valves, therefore, signs of failure will appear as a progressive reduction in this current long before the smallest change in the performance of the amplifier is detectable. For example, a fall of 20 per cent in anode current is easily readable, but would not affect the performance of the instrument.

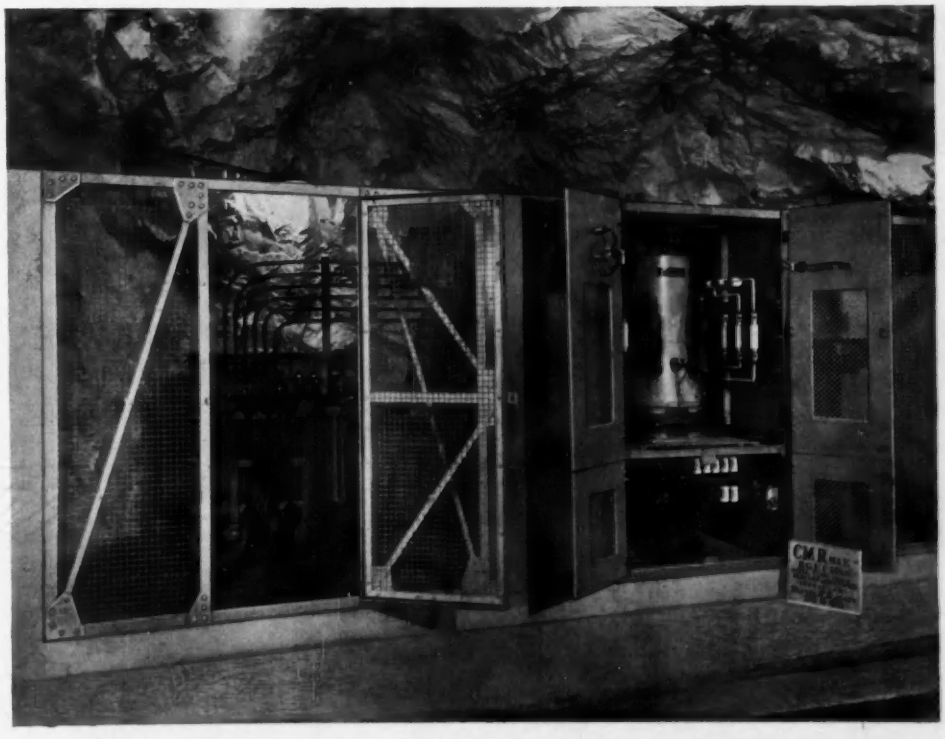
A milliammeter is provided (see fig. 1) with a switch by means of which it can be made to indicate the anode currents of the two amplifying valves. The regular use of this monitoring device would enable any deterioration in valve performance to be anticipated before errors became appreciable.

9. CONCLUSION.

The introduction of an instrument of this kind into the sphere of precision electrical measurements cannot be regarded as successfully accomplished without the satisfactory completion of a long period of rigorous practical trial. Such trials are now in progress in the G.E.C. Research Laboratories on the experimental apparatus shown in fig. 3. Should they prove favourable—and there is as yet no indication to the contrary—it would seem that the principle of using a stabilized amplifier with convenient monitoring may be used with confidence in even the most accurate A.C. measurements.

The extension of the voltage range of the present instrument up to 7,500 volts seems feasible, assuming special air-condensers to be used for the input-potentiometer. This will render it suitable for use with high voltage electric discharge tubes. The extension of the range downwards presents more difficulties since, with the existing number of valves, it would entail a reduction in accuracy and stability. With the addition of a further stage of amplification, however, a substantial increase in sensitivity should be available if required.

Rectifier Equipment in the Rand Mines.



An air cooled steelclad mercury arc rectifier equipment installed underground at the Consolidated Main Reef Mines, South Africa. The rectifier is rated at 120 amps. 500/550 volts D.C., the A.C. supply being 1950/2200 volts three phase. It is of interest to note that the rectifier is situated 6,000 ft. below the surface in an ambient temperature of 35 degs. C., and that the entire equipment was required to pass down a shaft 58ins. by 40ins.

A Small Hydro-Electric Plant in Chamba, Northern India.

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CHAMBA State is one of the oldest native principalities in Northern India, having been founded in the middle of the 6th century A.D. It is situated in the Western Himalayas, approximately two hundred miles from the Tibetan border. Within the State are three valleys which are separated by well-defined snowy ranges, and approach to the capital is made by means of passes over these ranges which vary in height from 8,000 to 17,000 feet.

Chamba, the capital, is 3,027 feet above sea level and stands on a plateau on the bank of the River Ravi, a tributary of which provides the motive power for the small hydro-electric station. It is 19 miles by the shortest route, over a pass approximately 9,000 feet high, from the hill station of Dalhousie where the only metalled motor road from the Punjab ends. When making the journey to Chamba by horse, you can stop the first night at a State Dak Bungalow at Khajiar, a small hamlet lying in a marg surrounded by pines and mountains. The next day it is not long before a view of Chamba is obtained from a place known as Monkey Point. This spot is some 8,000 feet above sea level on the upper pass and the view of Chamba is particularly pleasing, lying in the valley surrounded by snow-covered mountains. A panoramic view of Chamba from Monkey Point is shown in the frontispiece of this issue.

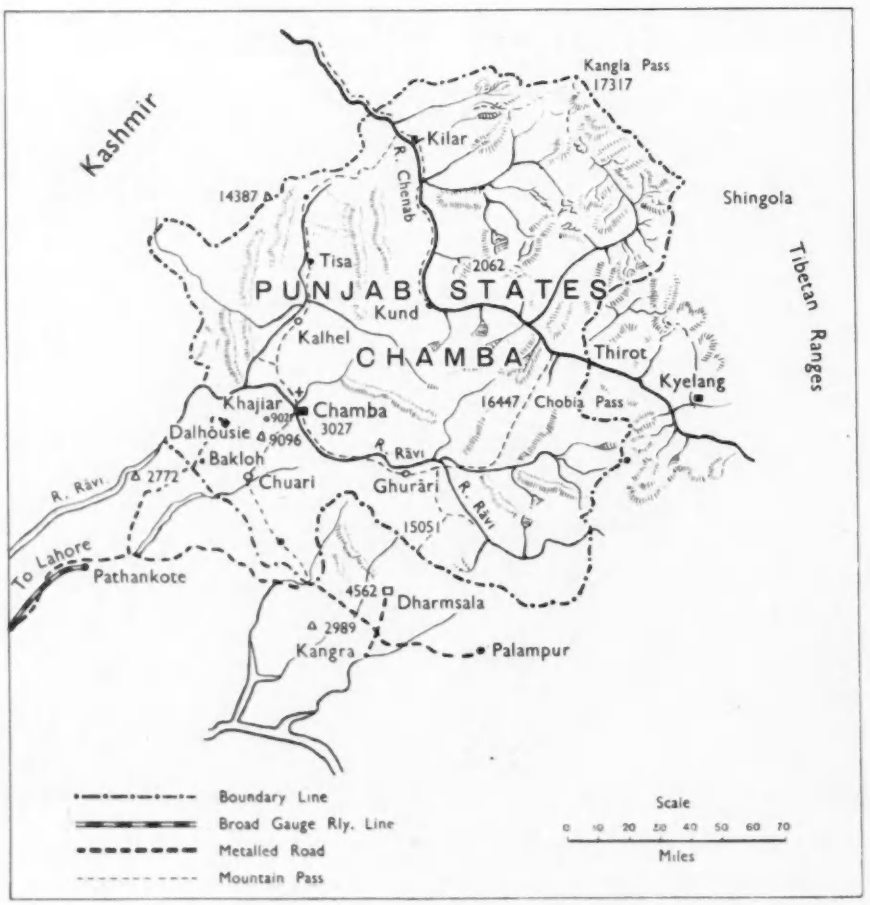

Fig. 1.—Map of Chamba State, Western Himalayas.

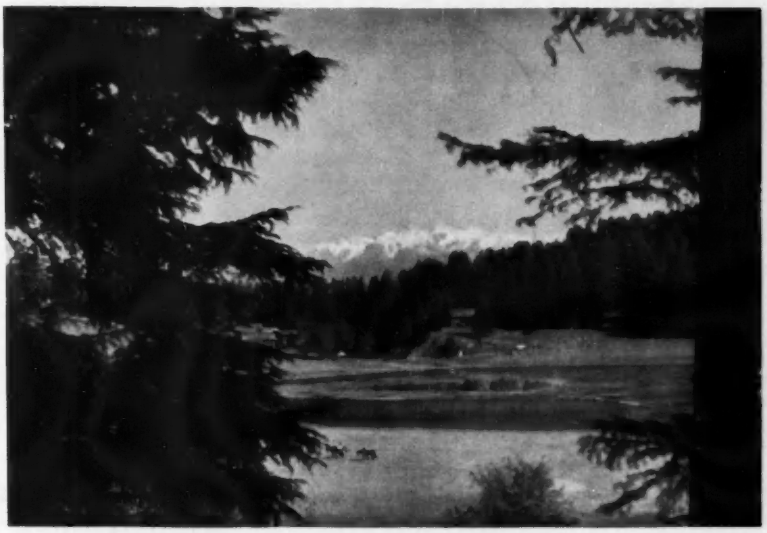

Fig. 2.—A State dak bungalow at Khajiar.

Mails to and from the State are carried by the local hillmen, known as dak runners, who operate in relays over the distance from Chamba to Dalhousie. It is remarkable how these runners get through in bad weather, particularly in mid-winter when the passes are almost blocked after heavy snowfalls. They always carry with them a long staff with a spear-head, attached to which are a few bells, and this affords the only protection from bear and panther which abound in the district.

There are some very interesting legends attached to the State, the best known of which concerns the Rani of one of the early Rajahs, in honour of whom a festival is held each year when all the women assemble on the hillside above the town, singing to the Rani who gave her life to provide water for Chamba. It is related that there was no good or convenient water supply and although the Rajah had a watercourse made from a stream, for some reason the water refused to enter the channel prepared for it. At that time this was ascribed to supernatural reasons and the spirit of the stream had to be propitiated. The Brahmins were consulted and decreed that the Rajah's son or the Rani must be sacrificed. The Rani, after many pleadings that she should be offered, wended her way up the hill accompanied by her maidens and was buried alive. The legend goes on to say that when the grave was filled in the water flowed immediately. To-day springs flow freely from the spot in the hillside where this festival is held.

The temples in the State are of two kinds, namely, hill temples and shikhara or plains temples, being associated with cults prevailing throughout the hills, some of them from ancient and aboriginal times. Some of them are known to date from 700 A.D., and many may be much older.

Chamba was the third state in India to have an electric power station installed, the work being completed in 1909. Mysore was the first, the installation being put into service in 1902 with a total capacity, as originally installed, of 4,000 kW; Kashmir was the second, with a total capacity installed of 4,000 kW, which was put into service in 1908. In Chamba the original sets installed consisted of two double flow 11-inch turbines with an output of 50 h.p. at 45 feet head of water. These have underwater bearings in each of which the main shaft is supported by three lignum vitae bearing blocks, and there are four such bearings to each machine. The governors are of the type known as King's governors and are driven by overhead gear and leather belts. The turbines are each coupled to a shunt wound interpole generator, running at a normal speed of 740 r.p.m. The generators were made to give an output of approximately 35 kW or 140 amps. at 250 volts. Until certain reconstructions were made to the headworks, the

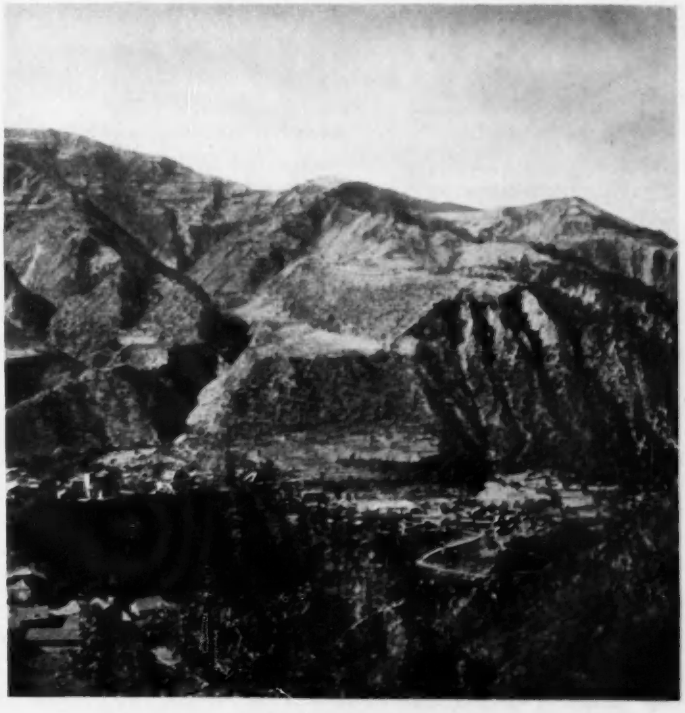

Fig. 3.—A distant view of Monkey Point from Chamba.

Fig. 4.—A Dak runner. The bells attached to the lower end of his spear are traditionally carried as a protection against evil spirits.

quantity of mud and silt carried down the flume by the water was so great, particularly in the monsoons, that replacement of the lignum vitae bearing blocks and clearing of the settling tanks were carried out daily, and the shafts had to be turned up once almost every five or six weeks. Due to the original sets having worn out, it is now not possible to get an output from each set of more than 25 kW, or 100 amps. at 250 volts. In addition to the generators there is a 10 h.p. oil engine set installed in the power house, capable of an output of 30 amps., but this was only intended to act as a standby, in cases of emergency, for the Palace or other important lighting loads. The two original sets had given some twenty-eight years' service but during that period their efficiency had fallen considerably and they could not be expected to operate continuously without frequent breakdowns.

The State authorities realised that the supply was totally inadequate for the present demand which consisted mainly of lighting for the bazaar, the Palace and State buildings, and particularly on festival days the peak load almost reached the maximum output the sets were capable of giving. For the future demand they had to take into account the occasion of marriage or other special State ceremonies, provision for additional street lighting and the operation of a new pumping plant which was to provide additional water for drain flushing and fire extinguishing, the latter requiring approximately 30 h.p. It was decided, therefore, to install a 100 kW turbine generator set in the available space in the power house and at some later date to remove the two existing sets and to replace them by a duplicate 100 kW set. The contract for the extension was entrusted to The General Electric Co., Ltd., who manufactured the whole of the electrical equipment and accessories and were responsible for the installation of the complete plant. The water turbine was manufactured and supplied by Boving & Co., Ltd., London.

The new set consisted of a 150 b.h.p. single runner casing turbine coupled to 100 kW generator and designed for a nett head of 45 feet with a normal speed of 750 r.p.m. and runaway speed of approximately 1,390 r.p.m. The runner is overhung on the free end of the turbine shaft and is carried by a bearing of the combined radial and thrust type mounted on the turbine cover and capable of taking any end-thrust that may occur, no underwater bearings being included. The turbine is complete with regulating shaft, governor with connections, pulleys and belt, stuffing boxes for the turbine and regulating shafts, and draft tube. With this type of turbine the whole of the available head is used, as the fall between the floor level and the tail

Fig. 5.—Old Hindu temples.

water level is recovered by means of a suction pipe or draft tube. The discharge from the runner is led to the tail race through the suction bend and draft tube which is arranged vertically, the bottom end of the draft tube being submerged in the tail race to approximately 9 ins. to provide a water seal. The

flow of water to the turbine is controlled by the automatic oil pressure servo-motor type governor, which can also be operated by a handwheel fixed on the governor.

The generator is of the protected type, shunt wound and having two endshield journal bearings and is coupled to one side of the turbine flywheel. It is designed to have an output of 100 kW at 230/280 volts at a speed of 750 r.p.m. and constructed to withstand an overspeed of 80 per cent. The generator is controlled by a flat back type switchboard panel complete with a 400 amp. double-pole air circuit breaker, reverse current relays, instruments and back of board hand-operated field regulator. A feeder panel similar to the generator panel with a 400 amp. double-pole air circuit breaker and necessary instruments was supplied. The two panels were made to line up with the existing switchboard which has the positive bus bar at the top and the negative bus bar at the bottom of the switchboard. In addition, spare parts for both the turbine and generator were supplied besides overhead line materials and hard drawn copper wire for increasing the capacity of the existing overhead line from the power house to the town.

The pipe line is of mild steel electric welded with an internal diameter of 30 ins. and suitable for a working pressure of 21 lbs. per sq. inch. It is opened out to 36 ins. bore at the inlet by a taper piece and has seven lengths of straight pipe of the spigot and socket type. There are three bends of the lobster-back type and an expansion joint of the sliding sleeve type suitable for 3-inch travel. A 30-inch throttle valve is fixed before the last bend to obviate its weight and pressure of water coming on top of the turbine casing.

With the introduction of the new set considerable alterations had to be made to the headworks and the flume. The excessive quantity of silt was due to the form of intake at the headworks and the quantity of water passing into the flume. The headworks is arranged on the outside bend of the river with the intake facing directly upstream, and consequently all silt, boulders or other material brought down by the river during the rainy season flowed into and blocked up the intake to the flume. Considerable improvement was made by the following alterations:—

(*a*) Construction of a groyne along the intake to divert the normal flow of the river away from the intake itself.

(*b*) The removal of a projecting boulder nose immediately upstream of the existing intake with the object of drawing the flow of water from the side of the river flow.

(*c*) The construction of a low weir below the intake to raise the normal water level to ensure an adequate supply of water to the intake and also to form a natural settling pond.

Whilst these alterations were being carried out the flow of water to the existing turbines could not be interrupted and this was ensured by damming up a section of the river with sandbags and large stones and diverting the flow. The weir was made up of stones and boulders taken from the hillside and enveloped by large steel wire cages, this complete assembly being finally cemented together into a solid mass leaving a smooth surface on the spillway and downstream face. The labour for this work and the reconstruction of the headworks was provided by Nepalese workmen, many of whom were on the Mandi hydro-electric scheme which provides electricity for the whole of the Punjab.

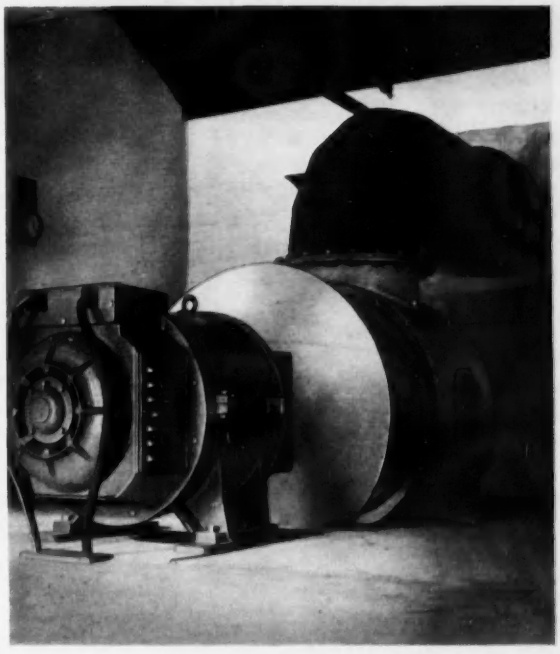

Fig. 6.—The turbine generator set in the power house. The set consists of a 150 b.h.p. single runner casing turbine driving a 100 kW 230/280 volt D.C. generator at 750 r.p.m., designed to withstand an overspeed of 80 per cent.

The flume is about 1,800 feet in length and follows a winding course along the hillside. The first 900 feet is in rock and the remainder consists of a masonry channel lined with cement. The first length of the flume for a distance of 300 feet is below the ordinary flood level and an intermediate opening is provided at 260 feet from the intake, so that in times of high flood, when the first portion is under water and covered with boulders, the water is drawn into the flume by a gate provided there. The first spillout is situated 850 feet from the intake just where the channel cut out from the rock ends. This offers a means of regulating the water flowing into the settling tanks which are constructed about 35 ft. ahead to give the correct amount of water necessary

for the turbines working at full load. There are two settling tanks, each 65 ft. by 16 ft., built in solid masonry and plastered with cement. They are easily cleaned by means of wooden chutes provided at each end, approximately 3 ft. wide, with grooves for planks which can also be used for regulating the supply to the flume channel. At the end of the second settling tank a strainer rack is arranged, from which all coarse materials can be removed. The flume from the settling tanks to the penstock chamber is built in lime masonry cement plastered. About 136 feet from the penstock chamber, a third spillout is arranged, built in cement masonry with the escape formed by a masonry channel cut through the rock and down to the stream which allows for the final regulation of the flow of water. A second strainer rack is arranged at the entrance to the penstock chamber to prevent leaves and coarser material going down the penstock pipes. The penstock chamber is solidly built with dimensions of 9 ft. by 40 ft. The supply of water to the turbines is regulated by means of locally made deodar wooden sluice gates arranged for hand operation.

With the introduction of the new set it was necessary to enlarge the section of the flume to give a flow of approximately 60 cusecs, allowing 35 cusecs for the new set and 13 cusecs for each of the existing turbines. The section was increased from 5 ft. 2½ ins. to about 5 ft. 4 ins. to accommodate the increased flow and was obtained by building upon the original walls of the flume to this height.

Further alterations were made on the tail race forebay and the penstock head spillway. The tail race which was originally directed upstream was blocked up, and a new cutting made, directed downstream, large enough to cope with the increased discharge with all three turbines operating. The existing pit and channel to the tail race had to be enlarged to accommodate the draft tube and to discharge

Fig. 8.—A section of the flume showing the first spill-out.

the greater quantity of water from the 100 kW set. For the latter reason, and to avoid too great a water level in the tail race channel, the weir was widened considerably.

Due to the sensitivity and rapid closing time of the oil pressure governor against fault conditions there was a danger of serious banking of the water on the penstock-head with probable flooding over the forebay on to the pipe track. A test was made by throwing off the full load, and although the rise in water level was not alarming the forebay walls were increased by one foot as a precautionary measure against a serious surge causing any damage. Further, the spillway at the penstock-head was enlarged to discharge the full quantity of water. The discharge flows down a masonry channel meeting the tail race below.

The transport of the plant to Chamba presented the biggest problem. The nearest railway station is at Pathankote in the Punjab, a distance of 72 miles, from where a regular motor service runs up a winding hill road to Dalhousie at a height of approximately 8,000 feet above sea level. It was learned from the State authorities, who were responsible for the transport, that only coolie or mule transport was available and that the machinery should be divided up so

Fig. 7.—Headworks and weir. The flood intake is being used as a spillout.

that the gross weight of any one piece or case would not exceed 1,200 lbs., as this was the maximum that could be handled effectively by twelve to sixteen coolies. Although it was possible to split up some parts, it was not practicable, owing to their bulk, for them to be carried by coolies, and the Superintending Engineer of the State had several robust wooden trolleys made up, measuring 11 ft. by 4 ft. overall, for transporting the heavy pieces of machinery. The total gross weight of the plant amounted to 12 tons, the turbine and governor being the heaviest and weighing 6 tons, electrical plant 3½ tons and

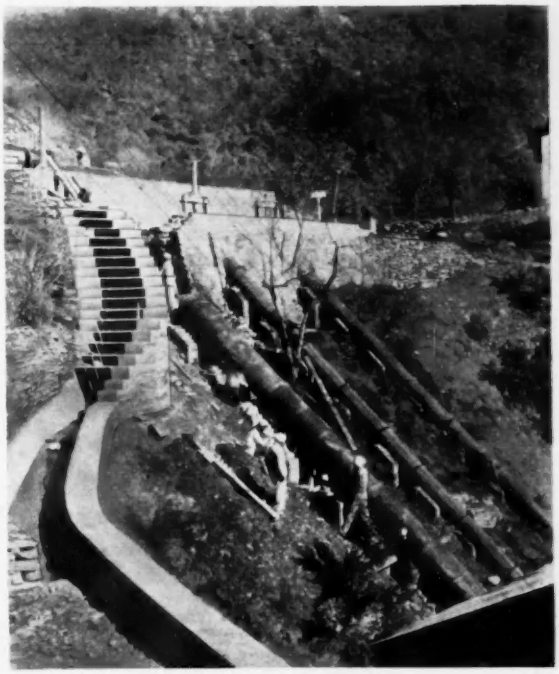

Fig. 9.—Erection of the pipeline: the discharge channel from the penstock.

the pipe line 2½ tons. The suction tube was not very heavy, but being 16 ft. long the transporting of it was very difficult. The generator frame was split into two halves, the bottom and top halves weighing approximately 1,570 lbs. and 1,180 lbs. respectively. The case containing the armature complete with bearings weighed approximately 1,600 lbs. The remaining cases containing the bearing brackets, rocker, brushgear, switchboard panels and turbine parts were all below 900 lbs. The pipe line was expressly ordered with seven straight sections to facilitate its transport and with a view to effecting a certain saving in transport costs.

There are three mountain roads connecting Dalhousie with Chamba, and of these the central one, called the Cohlri road (approximately 25 miles long), was chosen for transport purposes, as it had an easy gradient and was capable of taking all the wheel load. It was essential that the transport should be worked to a programme, as between November and March the passes are blocked for several days at a time due to heavy snowfalls, the upper pass being frequently impassable for several months.

Before the carriage of the machinery began many improvements to the road were necessary, as it was hardly 4 to 4½ ft. wide at many places, with sharp curves and large rocks and boulders projecting from the hillside on to the road. There was one particularly bad stretch between Dalhousie and a place called Talai, a distance of approximately 8 miles, and this had to be widened considerably and the rocks and boulders blasted. There were wooden cliff galleries along which the road ran and all of them were designed for pedestrian or mule traffic. These galleries had to be renewed, and big deodar trees were cut down and in some cases the whole tree spanned the road in order to stand the heavy weight of the machinery. A large number of retaining walls had to be rebuilt to make them strong enough for the heavy loads, as the wheel of the trolley was expected to pass over the edges and in several cases the walls gave way and had to be rebuilt. Wooden cantilever bridges over streams had to be strengthened with

Fig. 10.—Gangs of coolies transporting the turbine casing over the mountain roads on specially constructed wooden trolleys.

heavy wooden supports placed under the roadway girders, and struts under the flooring. In addition, 2-inch deodar planks were placed as an extra precaution under the trolley wheels when passing over these bridges. At one place the wooden cantilever bridge was not considered strong enough to take the heavy weight of the machinery and it was

Fig. 11.—Hauling the suction tube to site. The length of this pipe made its transport a matter of particular difficulty when negotiating the many sharp corners.

necessary to make a zigzag road down to the bed of the stream and up again to the other side of the bridge. The only means of crossing the river to the power house is by means of an aerial ropeway bridge, but this was not considered safe and a road was made down to the river cut from the sloping banks which had a gradient of about one in three, and a surface which was nothing more than loose stones. A bund had to be made across the river with stones leaving a small gap for the flow of the river, and an improvised bridge of wooden planks was erected on the day that the machinery was taken over. The narrow width of the road, which in some parts was bordered by precipices with a sheer drop of 2,000 ft., presented many difficulties; and at these places the gangs of coolies had to be divided into small parties, some pulling the trolley, some behind steadying it, and the remainder on the upper slopes of the hill holding on to the trolley by means of ropes in order to avoid any accident such as one of the wheels slipping over the edge of the precipice. The dimensions of the turbine casing and the suction tube made the transport most difficult in the narrow stretches of the road and at the everlasting U-bends and sharp corners. This kind of work was done at a great personal risk to the coolies, and the road repair gang had to be kept continuously at work following the trolley as it made its progress along the mountain road. The transport of only about half of the machinery had been completed when winter conditions started with heavy snowfalls, but in spite of this bad weather the transport was carried on unremittingly until the whole of the material was brought down safely to Chamba. The speed at which it was possible to transport the turbine casing, suction tube, the two halves of the generator and the armature was about 3 miles a day. The other lighter loads were moved at about seven miles a day, taking in all four or five days from Dalhousie to the power house at Chamba.

On the heaviest loads about sixty coolies were employed and on the lighter loads only twenty. They were usually divided into two parties, one at the front, hauling, and the other behind, steadying the load. The total number of coolies employed on the transport of the machinery, including the road repair gangs who also helped in the transport at the difficult places, amounted to about two hundred. An overseer accompanied all the cases, and the whole plant arrived at site in very good condition, with only a few breakages to glass and small fittings which would in ordinary circumstances be treated as negligible. A great deal of credit is due to these hill coolies for their dexterity and care in handling the cases in spite of the adverse circumstances.

The author is greatly indebted to Lt.-Col. H. S. Strong, President of the Council of Administration, Chamba State, for permission to publish this article with accompanying illustrations, and also to Gurditta Mal, Superintendent, P.W.D., Chamba State, for information relating to the transport and existing hydro-electric works.

Statistics and Engineering Practice.

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This paper was to have been read before the Institution of Electrical Engineers on 4th January, 1940, but the meeting was cancelled owing to the war. It was read and discussed before the Mersey and North Wales (Liverpool) Centre on 19th February, 1940. The authors have been awarded the Paris Exhibition (1881) Premium by the Institution.

In this paper the authors deal in greater detail with some of the more fundamental concepts briefly summarised in a previous article; they have in mind particularly the scrutiny of test data obtained in efforts made to control manufacturing processes, or to establish new procedures.*

The effect of chance on test values in the presence of inevitable variations in the properties of manufactured articles is discussed and illustrated in examples arranged in order of increasing complexity. The authors emphasise that the efficient use of statistical tools comes only by practice in association with sound technical knowledge.

(1) INTRODUCTION.

THIS paper is concerned with an elementary review of methods, based on statistical theorems, which can be used to help engineers handling data influenced by a multiplicity of causes to answer a question which arises continually in industrial practice. Concisely and in general terms it may be stated thus :—When knowledge becomes available by examination of samples drawn at random from a bulk, what conclusions can reasonably be drawn as to the character of the bulk?

Although statistics, in the sense of political survey, dates from the eighteenth century, the theory of statistics as a distinct branch of science did not begin to flourish until the last quarter of the nineteenth century. In England the work of Galton and Karl Pearson, followed by that of R. A. Fisher, laid the foundations of the application of theory to many fields of science. Research workers in biological sciences and in the agricultural industry were the first to turn these more recent advances to practical use.

Whilst individual workers, notable among whom was W. S. Gosset who wrote under the pseudonym of "Student," had used statistical methods in industry early in the present century, the first evidence of the systematic application of these principles to industrial problems was the publication of a series of papers by W. A. Shewhart and his colleagues in the *Bell Technical Journal* of 1922 and subsequent years.

In 1919, one of the present authors became interested in the appraisement of the quality of industrial products and in research and development work directed towards improvement of quality. The publication of the papers by Dr. Shewhart tended to confirm his growing conviction that statistical methods were necessary when studying data derived from factory tests, and also encouraged the regular application of statistical principles to this work. At this time there appeared to be no general use of these methods in English manufacturing industry if published work can be used as a criterion, although the introduction of tables of allowances on the life performance of electric lamps (1924) and the sampling clauses in coal specifications (1930, 1931) provide evidence of their use in industry. Greater interest was stimulated by a series of lectures given by Dr. Shewhart at University College, London, in 1932. Subsequently, the British Standards Institution formed a Committee charged with the following terms of reference :—

(i) To report on the application and use of statistical methods in standardization and specification of quality.

(ii) To draw up a short report which would serve to awaken interest in the application of statistical methods on the part of manufacturers and others concerned with problems of standardization and specification.

(iii) To consider what encouragement is necessary for the development of research on improved statistical methods and their application to industry.

(iv) To consider what steps should be taken to provide for co-operation with bodies in the United States of America and elsewhere instituted for similar objects.

Out of this activity also grew the formation of the Industrial and Agricultural Research Section of the Royal Statistical Society. The meetings of this Section have provided opportunities for technicians employed in industry to meet statisticians, and for statisticians to meet and discuss with technical

* G.E.C Journal, Vol. VIII, No. 3, August, 1937.

people the difficulties which arise in trying to apply statistical methods to the examination of industrial data.

That The Institution has been interested in this development is indicated by the publication of papers in its *Journal*[1,2] and in the *Students' Quarterly Journal*.[3] The Kelvin Lecture of 1938,[4] although not primarily directed to the consideration of industrial problems, called attention to the essentially statistical nature of life and science.

At the present time there are surely few industrial products which are not expected to conform with some standard of quality. Apart from measures taken to ensure that finished products will conform with quality standards demanded by consumers, standards are often imposed at various stages in the production process in order to reduce waste of material and, therefore, the cost of the marketable articles.

The maintenance of standards of quality invariably depends on engineers-in-charge taking action to diagnose and eliminate causes of changes in quality, the existence of which is suggested by the scrutiny of data derived from inspection and testing. On the one hand failure to recognize early any signs of a change in quality may have serious consequences and, on the other hand, much effort may be wasted if the suspicion that a change in quality has occurred is unfounded. Careful consideration will convince those experienced in the supervision of manufacturing processes that, apart from wasted effort, the staff of a factory will tend to develop a very undesirable mental outlook if repeated efforts are made to trace causes of trouble which do not exist.

A defeatist attitude may be adopted or changes in procedure may be made on inadequate grounds in order to provide some satisfaction for the efforts expended. Any such reactions will greatly add to the undoubtedly difficult task of maintaining a factory at a high pitch of efficiency with regard to the cost and quality of its products.

The main objects of the present paper are :—

To emphasize the essentially statistical nature of many technical problems, and the part that chance plays in many technical decisions.

To demonstrate the need for a technique which will give assistance in making deductions from test data.

To illustrate and briefly describe some of the more elementary methods, based on the theory of probability, which have been employed in industry.

To present them in an order which the authors' experience suggests will provide an easy mode of approach for an engineer employed in industry.

It is advisable to state in general terms how technical improvements and economies accrue to those industrialists who cultivate the statistical outlook, and who apply statistical methods to the scrutiny of technical data.

(i) Errors in judgment, arising from ambiguities due to the effect of chance, which lead to incorrect action, are reduced.

(ii) Development work involving reasonably large-scale production can be planned most economically and the results rightly appraised.

(iii) The efficiencies of specifications both for the internal control of factory materials and processes, and for normal commercial use, are improved when their compilation is guided by due appreciation of the errors which arise from chance causes.

(iv) Efficient and simple systematic methods of presenting data requiring daily scrutiny can be readily devised.

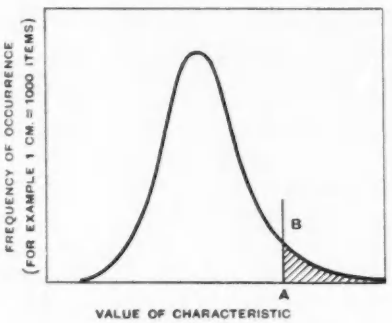

Fig. 1.—Frequency distribution.

(2) FREQUENCY DISTRIBUTION OF OBSERVATIONS.

When a large number of measurements are made on some dimension or quality of manufactured articles, it will generally be found that the observed values tend to be grouped in a characteristic manner. The frequency with which a value occurs diminishes as the difference of this value from a central value increases. The general arrangement of frequencies with which different values occur is known as the "frequency distribution" and may be illustrated by means of a frequency diagram of the type shown in fig. 1. If a conception can be formed of the frequency distribution of the observed values which could be obtained by the examination of a large number of articles, then some estimate can be made of the likelihood of obtaining any specific value if an article is chosen at random from the bulk. Thus, in the diagram, the probability* of a random observation having a value greater than that corresponding to A is the area under the curve to the right of AB expressed as a fraction or a percentage of the total area under the curve.

Published theoretical work has been largely associated with the properties of the Normal or

* It is advisable to point out that the authors use the word "probability" here and throughout the paper in the sense of "likelihood based on proportional frequency of occurrence."

Gaussian distribution, the formation of which was illustrated by Professor Born in the Kelvin Lecture of 1938.[4]

The Gaussian distribution has the property of being defined by the value of the mean and another quantity which is called the "standard deviation." The standard deviation is a measure of variation and is the root-mean-square of all the deviations of the individual values from the mean value. It is, therefore, analogous to the R.M.S. value of an alternating current flowing in a circuit. When a mean value and a standard deviation are associated with a large amount of data, they are usually represented by the symbols $\overline{X}$ and σ, but when associated with samples by $\bar{x}$ and s, respectively. The standard deviation is sometimes expressed as a proportion or percentage of the mean. It is then known as the "coefficient of variation."

For the Gaussian distribution, tables have been calculated and published[5] giving the area of the curve lying outside various deviations from the mean expressed as multiples of the standard deviation. It is convenient to refer to deviations so expressed as "standardized" deviations. By using these tables, if the mean value and standard deviation of a Gaussian distribution are known, or can be inferred, the expected proportion of individuals within any desired range of the scale of measurement can be estimated.

For example, the proportion of observations having a value

less than $\overline{X}-\sigma$ or greater than $\overline{X}+\sigma$ is 0·159
less than $\overline{X}-1{\cdot}7\sigma$ or greater than $\overline{X}+1{\cdot}7\sigma$ is 0·045
less than $\overline{X}-1{\cdot}96\sigma$ or greater than $\overline{X}+1{\cdot}96\sigma$ is 0·025
less than $\overline{X}-2{\cdot}3\sigma$ or greater than $\overline{X}+2{\cdot}3\sigma$ is 0·011
less than $\overline{X}-3{\cdot}09\sigma$ or greater than $\overline{X}+3{\cdot}09\sigma$ is 0·001

These proportions are illustrated in fig. 2.

Should the frequency distribution not be Gaussian it should be noted that the standard deviation is still a measure of the spread or variability of the data, but the proportions associated with particular standardized deviations would differ from those given above.

The engineer contemplating the use of established theoretical relationships is confronted with two principal dangers. On the one hand he may apply these theoretical relationships without any appreciation of the limitations which may be imposed because the distribution with which he is concerned departs from the Normal or Gaussian. On the other hand, in the absence of knowledge of the distribution, he may hesitate to use statistical principles at all.

However, experience shows that, where reasonable care is taken to control the processes of manufacture and measurement, there is a marked tendency for the distribution of observations to approach the Gaussian form. Many examples of such distributions are available, and a few are illustrated in figs. 3—15.

In those cases where the distribution is not Gaussian, despite the fact that there is reasonable control of manufacturing processes, the general "single-hump" shape of the distribution is retained, but there is lack of symmetry (*e.g.*, fig. 7). However, it can be shown that if the average value derived from tests of samples as small as, say 5 articles, be utilized, the distribution of these average values will show an increased tendency to be Gaussian whatever the nature of the distribution of the individual values. Hence it is clear that some of the theoretical relationships derived from the properties of the Normal or Gaussian distribution can be safely used in approaching practical problems if, in the early stages, attention is concentrated on the average results derived from small samples rather than on the individual values. As experience is gained, the limitations imposed by any assumptions made when interpreting data derived from samples become more readily appreciated and methods available for dealing with individual observations can be utilized without serious risk of error.

(3) CONTROL CHARTS.

(a) Chart for Mean Values.

There is no doubt that Shewhart's conception of a control chart, on which routine test data are plotted against limits, forms the best introduction to the application of statistical principles to industrial

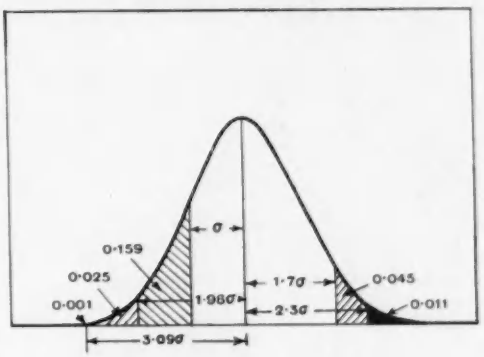

Fig. 2.—Gaussian distribution.

problems. In its simplest form it assumes that some measured quality of a manufactured article is represented by a frequency distribution similar to that already referred to, and that this distribution is fixed when the industrial processes proceed under normal conditions of control. As already explained the spread in such a distribution can be measured by the standard deviation and, if the limits Shewhart chooses (three times the standard deviation) be placed on either side of the central value it is clear from many of the illustrations already given that observations outside these limits should occur on very rare occasions. With this background the occurrence of an observation outside these limits is an indication

to the engineer that something may be happening which suggests a lack of control of the materials or processes.

The association of a value of probability with limiting values displaced from the average value by a multiple of the standard deviation depends on the

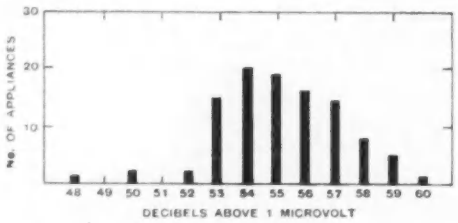

Fig. 3.—Radio interference from small domestic appliances.

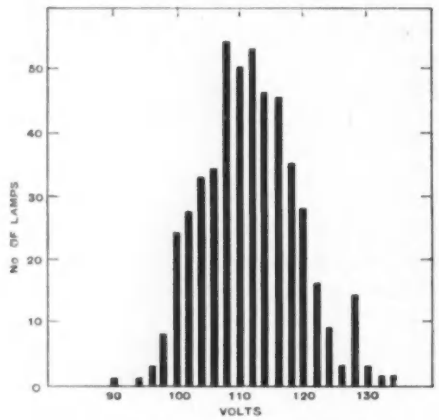

Fig. 4.—An electrical characteristic of 400-watt mercury-vapour discharge lamps.

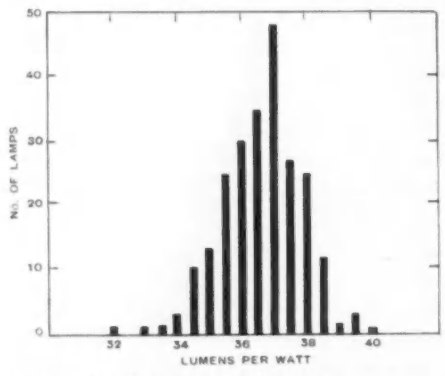

Fig. 5.—Initial efficiency of 250-watt mercury-vapour electric discharge lamps.

distribution of values approaching some established form, for example the Gaussian as illustrated in column, 1, page 197, and in fig. 2. Bearing in mind what has been stated at the end of the previous Section regarding approach to the Gaussian form, it is advisable, when making contact with statistical technique for the first time, to deal with the variation among average results for samples of, say, 5 to 10 individuals.

Even when samples are selected from the same bulk the average results for the different samples will inevitably differ because of the existence of variation in values for individual specimens. Hence, when studying the variation among the average results of samples, with a view to deciding whether they could

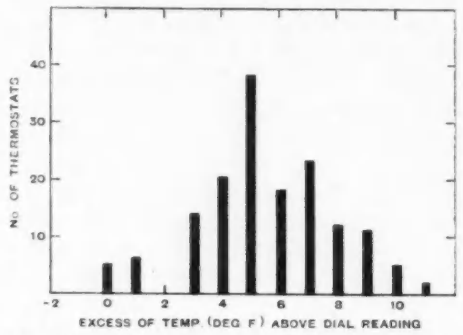

Fig. 6.—Errors in adjustment of thermostats used to control electrical heating apparatus.

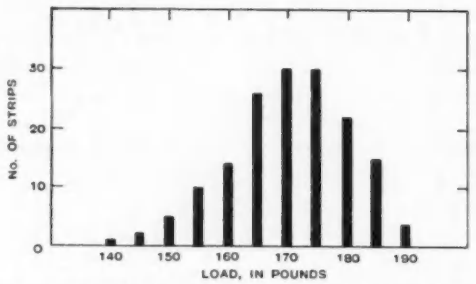

Fig. 7.—Breaking load of strips of cotton fabric.[6]

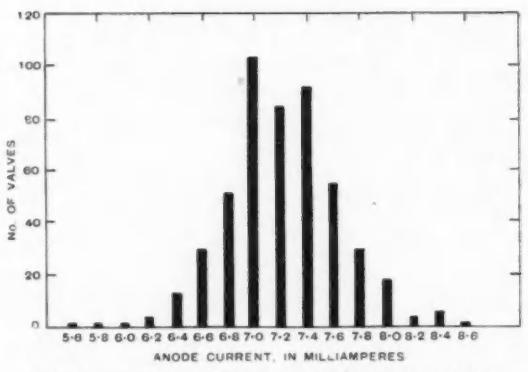

Fig. 8.—An electrical characteristic of receiving valves.

have come from bulks having similar average qualities, allowance must be made for the variation which would exist even if the samples were drawn from the same bulk.

Irrespective of the form in which the individuals are distributed, the standard deviation of the mean values of samples containing n individuals is given by $\sigma/\sqrt{n}$, σ being the standard deviation of the individual results comprising the bulk from which

the samples are selected. Thus, if the data under consideration are the average results derived from tests containing 4 or 9 individuals, the standard deviations of these average results will be respectively one-half or one-third of the standard deviation of the individual results. Further, the average value

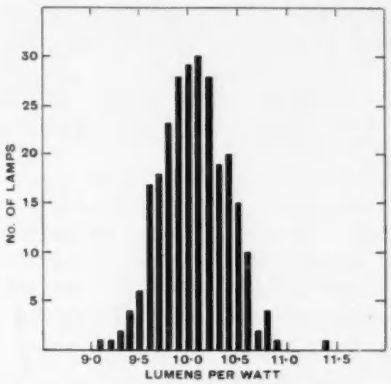

Fig. 9.—Initial efficiency of 40-watt electric incandescent lamps.

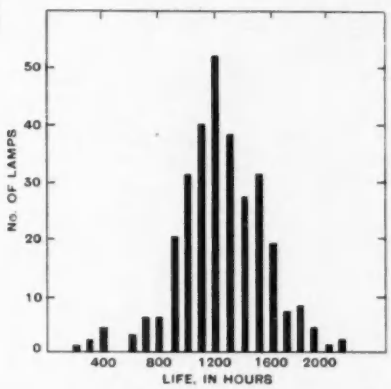

Fig. 10.—Lives of 100-watt electric incandescent lamps.

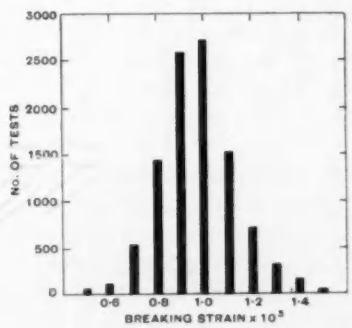

Fig. 11.—Breaking strain of glass tubing.

of all the sample means will approximate to the average value of the individuals in the bulk. Hence, since an approach to the Gaussian form can be assumed, standard multipliers can be used to calculate limits within which the means of samples will be expected to fall on, say 19 out of 20 or 499 out of 500 occasions, provided the bulk from which they are drawn has a specified mean value and standard deviation. For example, the means of samples of n individuals selected from a bulk with a mean value $\overline{X}$ and standard deviation σ will fall

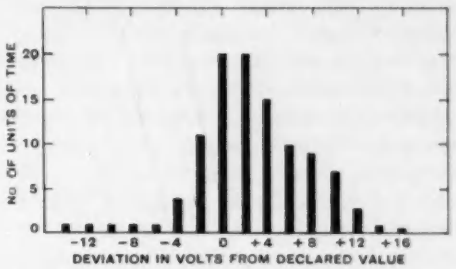

Fig. 12.—Voltage distribution on a network.[7]

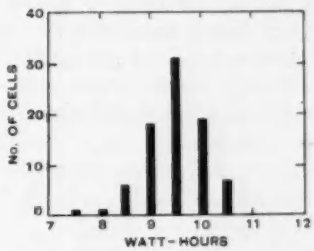

Fig. 13.—Watt-hour capacity of primary cells.

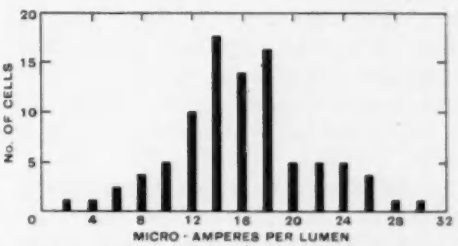

Fig. 14.—Sensitivity of photo-electric cells.

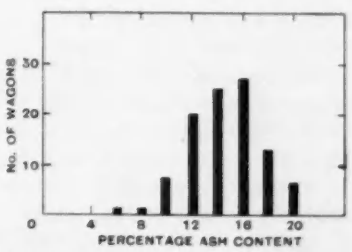

Fig. 15.—Percentage ash content in wagons of coal.[8]

19 out of 20 times between limits $\overline{X} \pm 1{\cdot}96\sigma/\sqrt{n}$
499 out of 500 times between limits $\overline{X} \pm 3{\cdot}09\sigma/\sqrt{n}$

The values of the mean results of samples relative to such limits will indicate whether it is likely that the samples have come from bulks whose quality values

are consistent with those on which the limits are based. If data for samples selected from a regularly produced product are found to be consistent with the limits, the quality of the product can be said to be controlled. Such limits may therefore be known as "control limits" and the graphical representation as a "control chart." In effect a controlled product may be said to be one in which variation is influenced only by "chance causes," "chance causes" being those inherent factors always present in a manufacturing procedure and causing variations which, owing to lack of knowledge or on the score of impracticability, it is not possible to eliminate.

Many of the applications of statistical technique to industrial problems are concerned with deciding whether the variability of mean results is consistent, within defined limits of probability, with the assumption that only "chance causes" are operative, or alternatively of indicating, with a measure of probability, that on a particular occasion a cause of variability other than "chance causes" was operative.

Causes of variability associated with particular occasions have been termed "assignable causes" by some writers. These are those extraneous or occasional factors which may affect the product. In industrial practice, when the presence of such factors has been detected statistically, it is the duty of the technician to trace their precise origin and then to eliminate them.

The control chart serves to indicate the presence of extraneous causes of changes in the quality of a product, and after attempts have been made to eliminate them it serves to demonstrate whether or not they have been successful.

The construction and use of a control chart for mean results will be illustrated by utilizing the data given in Table 1.

In order to test whether the data under discussion are consistent with a bulk of fixed quality, the mean and standard deviation of such a bulk are required. They may be known from previous experience of what is satisfactory quality. Alternatively, on the hypothesis that the manufacturing processes are controlled and that only chance causes are operative, reasonable estimates can be made from the results of 5 to 10 samples each containing 5 to 10 individual results, provided that each of the samples is selected from a range of product which is subject to the operation of all the chance causes.

The variability in the average results may arise from :—

(*a*) A system of chance causes which affect the results for all samples, and

(*b*) Assignable causes which affect the results of samples differentially because they are operative occasionally.

In order to obtain a measure of the variability arising from (*a*), the standard deviation is calculated from the differences of the result for each individual lamp from the average value for its appropriate sample of 5 and not from the average value of all samples. By this procedure the effect on the magnitude of the standard deviation of any variability arising from (*b*) is not included, whether or not such variability exists.

Assuming that the data given in Table 1 are those obtained when recording was initiated, the average result and the standard deviation are calculated* from the first 10 samples of lamps. The average is found to be 1,430 hours and the standard deviation 575 hours.

TABLE 1.

LAMP DATA: LIFE OF 25-WATT LAMPS AT RATED VOLTAGE (5 IN EACH SAMPLE).

1st Period			2nd Period		
Sample No.	Mean	Standard deviation	Sample No.	Mean	Standard deviation
1	2,080	464	1	1,295	440
2	1,202	575	2	2,005	435
3	1,295	422	3	2,445	580
4	1,190	406	4	1,900	345
5	987	653	5	2,570	290
6	814	487	6	1,980	510
7	2,356	521	7	1,990	445
8	1,280	639	8	1,990	315
9	1,280	318	9	1,715	385
10	1,818	555	10	1,650	460
11	1,456		11	1,935	
12	1,646		12	1,760	
13	1,060		13	2,175	
14	1,040				
15	1,234				
16	973				
17	1,128				
18	1,495				
19	2,472				
20	1,047				
21	1,465				
22	1,289				
23	1,650				
24	1,929				
25	1,199				
26	983				
27	1,148				
28	1,642				
29	1,714				
30	1,650				
31	970				

These values are estimates of the average and standard deviation of the bulk product represented by the samples. Now if no other causes of variability appear which would affect differentially the results from sample to sample, then the average results for the samples of 5 lamps should conform to the statistical relations which govern random selections from this bulk. When the standard deviation of individuals is 575 hours, the standard deviation for the average results on samples of 5 individuals is $575/\sqrt{5} = 257$ hours.

* For method see B.S.S. No. 600—1935, p. 83, para. (*a*).

Hence, considering the average life for the samples of 5 lamps, since the distribution of such averages will be approximately Gaussian in form, only 1 result in 20 should lie outside the limits $1,430 \pm 257 \times 1{\cdot}96$, or 930 and 1,930, and only 1 in 500 results should lie outside limits of $1,430 \pm 257 \times 3{\cdot}09$ or 630 and 2,220, provided the data are consistent with the samples being selected at random from a bulk or bulks with average = 1,430 hours and S.D.* = 575 hours.

Whether or not the test data conform with this requirement can readily be seen by constructing the control chart illustrated in fig. 16, where the full line represents the average of the first 10 samples and the dotted lines AA′ and BB′ the 1 in 20 and 1 in 500 limits, respectively.

Referring to the chart, where a further 21 observations are plotted, it will be noted that the data conform reasonably well with the limits. This provides general evidence that the lamp life is not greatly inconsistent with the assumption that the variability in the average values is due, in the main, to the chance causes.†

The control chart, with its limits based on statistical concepts, thus provides a scale associated with probability against which to gauge whether differences in test results are due to the existence of differences in quality or to chance causes. An illustration will now be given of how the chart will indicate whether a significant improvement of the product has resulted from a deliberate change in manufacturing procedure.

In general, the quality aim of lamp-manufacturing development is to reduce variability and if possible to increase the average efficiency of light production without reducing life. For obvious reasons it is not practical to attack the second portion of the problem directly but rather first to improve life and subsequently to improve efficiency. In 1933, when the above data were obtained, preliminary investigations, involving laboratory work followed by small-scale production, had indicated that a modified manufacturing procedure would lead to an improvement in lamp life. Regular production was undertaken and data derived from the routine testing became available. Do the later results confirm that an improvement has been effected and the production controlled at the improved level?

In Table 1 results for the first 13 batches of 5 lamps, selected at random from lamps made under the new condition, are given and are shown plotted on the diagram, which is broken to illustrate a lapse of time between the two sets of data.

It is obvious that the new data depart markedly from the statistical limits derived from the old data, and an improvement in the length of life is demonstrated.

Adopting the procedure described above, new limits CC′, DD′, have been calculated from the first 10 batches of 5 lamps selected from the new production. After some uncertainty amongst the first samples it was found that the quality of the production has settled down at the higher level, a fact illustrated by the few results plotted. Here, besides fulfilling the function of indicating whether the quality of the product is stable, the control chart has been used as a yard stick. It has provided a means for demonstrating that the improvement indicated by tests on samples actually exists and is not due to the chance of selection from an unchanged product.

A discussion of a similar example is given in B.S.S. 600,[9]* and a careful study of this would give valuable assistance to anyone making his first excursion into this field. Many similar examples will be found in Shewhart's book,[10] and another is treated in considerable detail in a paper by Dudding and Baker,[11] which endeavours to answer questions that are frequently raised by the production engineer.

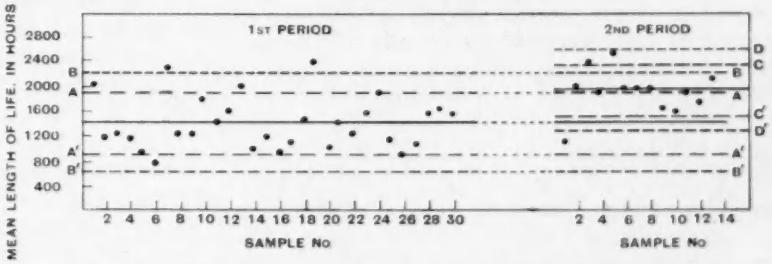

Fig. 16.—Control chart for means of samples of 5. Lamp life data.

In the above example the control limits have been based on the average and standard deviation of 10 samples of 5 individuals. In actual practice, as more data become available, it is desirable to re-calculate $\bar{X}$ and σ either to include more data or from another section of the data. Further, reconsideration of the size of each sample may be necessary. Appropriate action cannot be specified, for it will depend on the nature of each problem. The cost of testing, the degree of variability exhibited by the product, and the sensitivity required in the study of changes, will have to be taken into consideration. In this connection, some comment is necessary on the limits chosen. Shewhart, throughout his book, uses limits of 3 times the standard deviation from the mean, which

* Standard deviation.

† It will be seen that the results are not statistically consistent with the assumption that no variability exists other than that due to the chance causes. This actually led to an investigation of the cause.

* The pamphlet is concerned almost entirely with the statistical ideas and methods relating to control charts and gives a very full and practical account. It can be recommended as an introductory handbook to the general study of statistical methods.

corresponds to 2 to 3 observations per thousand in the tails of a Gaussian distribution. In the example described above limits corresponding to 1 in 20 and 1 in 500 respectively in the tails are shown. Whilst the wide limits, usually associated with Shewhart's choice, will increase the certainty that results lying outside them indicate the existence of change, they also increase the chance of appreciable changes in the level of the quality of the product being unobserved.

In order to minimize this difficulty limits should be utilized which correspond more nearly to the magnitude of any change it is desired to detect. This can be achieved by increasing the number of individuals in a sample. However, in most cases, this procedure would prove impracticable and uneconomic. An alternative frequently adopted is to use closer limits corresponding to a higher probability than 1 in 500, such as the 1 in 20 limits included in the above illustration. The use of these narrower limits will introduce a 1 in 20 risk of a false indication that a change in quality has occurred, and in order to avoid the effect on factory staff (referred to on page 196) the occurrence of a point outside the control limits should be treated as a warning only. Serious action should be taken only after tests on additional samples confirm the existence of a fault or after a number of such points have occurred close together. The narrower limits, however, will give an indication of changes of quality which might be missed if only the wider limits were used. For example, in the above problem, if the quality of the product were to fall 40 per cent, owing to some defect developing in the manufacturing procedure, the chance of an observation falling outside the wider limits (1 in 500) illustrated is still of the order of 1 in 6 tests, whereas the chance of it falling outside the narrower limits (1 in 20) is of the order of 2 in 3. That is, using the wider limits there is considerable risk of the change in quality being overlooked, whilst using the narrower limits there is a high probability of detection. These differences are of considerable practical importance when watching a continuously operating manufacturing procedure.

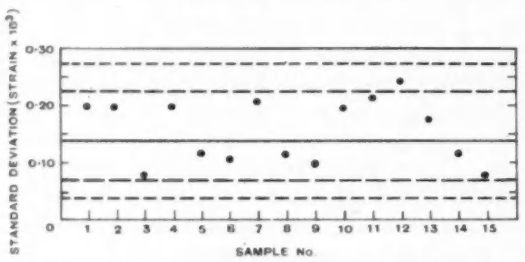

Fig. 17.—Control chart for standard deviations. Breaking strain of glass tubing (8 in sample).

The discussion of a specification problem given later in the paper will assist in making clear how the differentiation of levels of quality determined by sampling involves necessarily the concept of probability. This reference to choice of control limits is only made to indicate the kind of problem which comes up for consideration. Every case can be considered on its merits, and those responsible for making decisions can be fully aware of the risks that are being taken and appraise their technical and commercial significance.

(b) Chart for Variability.

Control charts, similar to that described for the means, can be made for the standard deviations or for their associated measures of variability, the coefficients of variation of samples. Such charts will not be discussed in detail; their construction and use are described in B.S.S. No. 600.[9]

An example of a control chart for standard deviations is given in fig. 17. The data are the standard deviations of some of the separate samples of 8 specimens which were included in the general distribution of strengths of glass tubing, as shown in fig. 11.

(c) Chart for Proportion "Defective."

(i) Use of binomial distribution.

Frequently when studying the quality of a product the chief interest will reside, not in averages or general variability, but in the proportion of articles which lie above or below a specified limit of quality. In statistical literature this proportion is usually referred to as "defective," but it must be understood that whilst the term may correctly describe the articles in question it will frequently be used when referring to an article whose quality is merely below a certain standard.

When material can only be classified "good," "bad," "broken," or "unbroken," it becomes necessary to know how the chance of selection can affect the proportion of a sample falling within any class. A similar problem can arise when considering a characteristic measured on a continuous scale if the interest resides in the proportion above or below some specified limit value.

If a sample of n articles be chosen at random from a bulk containing a proportion p of defective articles, it is to be expected that the proportion of defectives in the sample will not always be exactly p, but will vary about p owing to the effect of chance in sampling.* It will suffice to state here that, if repeated samples of n are chosen from a bulk with proportion p defective the proportions of occasions on which 0, 1, 2, etc., up to n, defectives will be observed are given by the terms in respective order of the expansion of the binomial $(q + p)^n$, where $(q + p) = 1$. For example, a fraction q^n of the samples will contain 0 defectives, a fraction npq^{n-1} will contain 1, etc. On the basis of this relationship control limits can be deter-

* Professor Born illustrated the underlying principles governing this effect and they are discussed in the paper read before the Scottish Students' Section.[5]

mined for the number or proportion of defectives in a sample within which results should fall, say 19 out of 20 or 499 out of 500 occasions if the samples are consistent with being randomly drawn from a bulk with a specific defective proportion.

Examples are given in fig. 18 of the frequency of occurrence of errors greater than 0·5 per cent in the diameter of fine tungsten wires. This is used as one of the controls on this dimension (errors greater than 1·5 per cent rarely occur). The diameters of the wires are of the order of 0·0005in., and 0·001in., and about 100 and 400 individual wires form the sample in the two cases respectively.

Limits are calculated from the binomial expansion using a bulk proportion p of defectives based on the results observed, and it is seen that, in the case of the smaller wires, the variations are likely to be due to chance. For the larger wires a real improvement is indicated. Further data, which have been included in the diagram, show that these indications are correct.

(ii) Use of mean and standard deviation.

In the previous Section [(*c*) (i)] the appraisement of the proportion defective in the bulk has been approached by studying the actual proportion observed defective in the sample. This will often be the safest method to adopt but, in small samples, it may have certain disadvantages. One may mention,

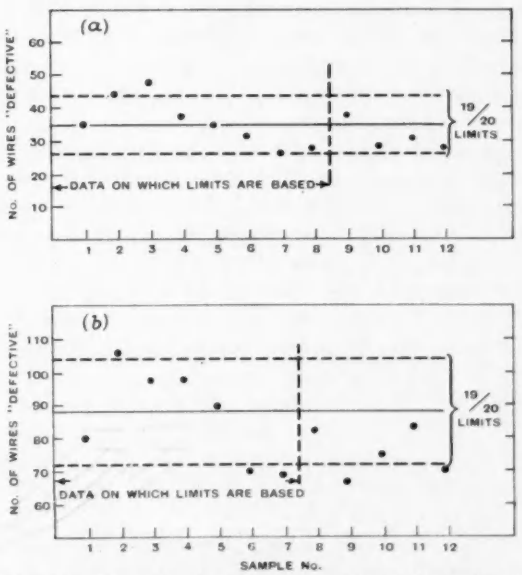

Fig. 18.—Control chart for proportion "defective." Numbers of tungsten wires with errors in size greater than 0.5 per cent.

(*a*) 15-watt and 25-watt wires (80 in each sample).
(*b*) Other tungsten wires (400 in each sample).

for instance, the difficulties which arise owing to the discontinuous nature of the binomial distribution. (This point is emphasized by the example discussed in the next Section). It is natural, therefore, to ask whether, in the case of a continuously variable quantity, a better method of obtaining information about the proportion defective in a bulk could be utilized which would make use of the observed mean ($\bar{x}$) and standard deviation (s) of the sample. Whether this is so or not depends on what assumptions can be made about the form of the distribution which is being sampled.

Thus, if a product is known to follow the Gaussian distribution, and any changes likely to occur consist only in movement of the distribution bodily without any attendant alteration in the true standard deviation (σ), the sample mean will be the best statistic to

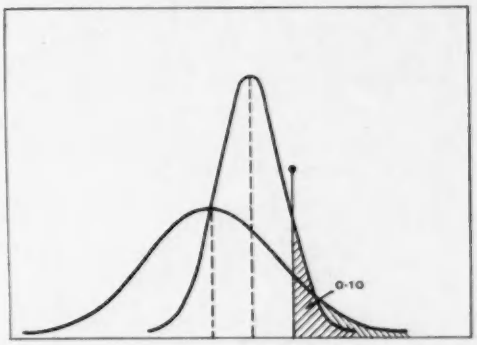

Fig. 19—Gaussian distributions with 10 per cent of area above limit.

study, whether information is wanted about the mean of the population or about the proportion lying beyond any defined limit whatever. However, if the mean position and the spread of the distribution may change whilst its mathematical form is likely to remain constant, then a function combining the mean and standard deviation of the sample is the best to use.

For example, referring to what has been said regarding the Gaussian distribution, the proportion of items beyond a limit L depends only on the "standardized" deviation of L from the mean; that is, on the ratio of the deviation of L from the mean to the standard deviation.

Thus, for instance, all Gaussian distributions which have $(L-\overline{X})/\sigma = 1{\cdot}28$ will have 10 per cent of the values greater than L. The quantity $U=(L-\overline{X})/\sigma$ will, therefore, serve to connect $\overline{X}$ and σ in various bulks which satisfy the condition that they shall have the same proportions above the limit L (see fig. 19). If a sample is available then $u = (L-\bar{x})/s$ can be calculated from the mean ($\bar{x}$) and standard deviation (s) of the sample as an estimate of U. For a given value of U and size of sample n, it is possible to calculate control limits, corresponding to any desired probability, within which the sample values of u should fall, providing the samples conform with bulks for which U is constant. These limits, for example, could be for probabilities of 1 in 20 and 1 in 500 of results falling outside them. A discussion

of the method of calculating these limits, together with a fuller discussion of their application, is given in a paper by Jennett and Welch[12] [see also Ref. (13)].

A control chart for u is shown in fig. 20, which relates to lamp life data. The objective is to indicate that the proportion of lamps with lives below approximately 700 hours is 10 per cent or less. (This is, of course, the same as controlling the proportion above approximately 700 hours to 90 per cent or more.) The data here are quite consistent with the hypothesis that 90 per cent or more of the product have a life exceeding 700 hours.

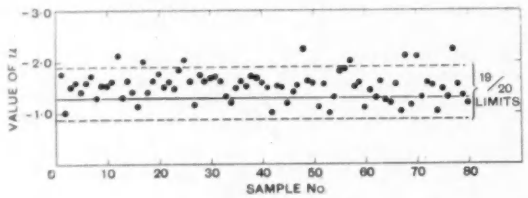

Fig. 20.—Control chart for $u = (703 - \bar{x})/s$ (30 in sample). Lamp life data.

$\bar{x}$ = sample mean.
s = sample standard deviation.

It must be emphasized that the situations in which U can be employed for the purposes of controlling the proportion below a specified limit of quality are confined to those where knowledge of the product suggests that the distribution can be relied upon to be reasonably stable in form, not only in the centre of the distribution but also out towards the tail being studied. It is dangerous to attempt to choose a limit such that, under normal conditions, the chance is exceedingly small that an individual should lie outside it, *i.e.*, in effect to extrapolate a distribution into regions where an assumption as to its exact form is necessarily precarious, being based on insufficient data. Thus, it is not being suggested that measures such as U based on a general distribution of values have a direct bearing on the engineer's "factor of safety." Such factors are designed to meet the occurrence of very rare events which are associated with causes extraneous to the system of continuously operating chance causes which give rise to the general variability. When "factors of safety" are referred to as "factors of ignorance" the description may be a really true and sound one, for the engineer who uses them does so because he humbly admits that there are limitations to his experience. Anyone who extrapolates his experience to indiscreet limits may be exhibiting a measure of ignorance dangerously exceeding that with which "factor of safety" is disparagingly linked.

The above method, which deals with the proportion defective by considering the mean and standard deviation of the sample, is likely to have a valid application for moderate excursions into the tail of the distribution, but, of course, what is "moderate" will depend on the actual problem under consideration.

(4) SPECIFICATIONS INVOLVING SAMPLING.

Specifications are usually designed to establish a standard of quality agreed between manufacturer and consumer, and contain some criterion by which it is proposed to decide whether or not materials offered conform to this standard. The criterion may be defined in many ways, in terms of a mean value, or the percentage falling outside specified limits, etc. It will be clear from what has already been described that, if sampling is involved, then whatever the specification criterion, samples, chosen from bulk supplies whose measures of quality correspond to widely different values of the mean and spread in the characteristic specified, may yield the same value of the specification criterion. That is, the influence of chance is such that a single sample may lead to material being accepted which is below the standard the specification was designed to establish and, contrarily, chance may lead to the rejection of "satisfactory" material.

The probability of material being rejected by the specification will depend on the actual level of the quality characteristic concerned relative to the level required by the specification, and on the degree of variability which would be experienced by repeated sampling from that material in accordance with the specification. The variability referred to here will depend on the variability exhibited by the material with respect to the characteristic measured and on the form of the specification.

Assuming that a limit value for a quality characteristic of a sample is defined to determine whether a bulk supply is accepted or rejected, then clearly, if the quality characteristic of a bulk supply approximates to the value defined for a sample, the material will have about a 50 per cent chance of acceptance or rejection. For example, if a product is rejectable when more than 10 per cent of a sample is "defective," then a bulk product containing 10 per cent "defectives" will have about a 50 per cent chance of acceptance or rejection, because if sampling were repeated many times one half of the samples would contain less than, and the other half more than, 10 per cent "defectives."

As the quality level of a bulk supply is raised above the sample criterion the chance of acceptance is increased and the chance of rejection reduced. On the contrary, as the quality level of a bulk supply falls, so the chance of acceptance is reduced, and of rejection increased. Hence, when considering the specification criterion for a sample there is essentially some conflict between the points of view of manufacturer and consumer. If the assumption be made that the manufacturer knows the general level and spread in

quality of his product, he has then to decide what chance he can take of having material of his normal level of quality rejected. He will, in general, desire the specification criterion for a sample to represent a level of quality lower than that he can maintain, in order to reduce his risk, say, to 1 in 20 or lower.

However, if the consumer appreciates the implications of a sampling specification, he will realize that this lower level of quality represented by the sample criterion will, of necessity, mean that there will be appreciable risk that he will accept material lower in quality than that of the manufacturer's product considered above.

Hence, once a sample criterion is fixed it will divide material concerned into three general classes :—

(i) Material of high quality which will be rejected on rare occasions.

(ii) Material of low quality which will be accepted on rare occasions, and

(iii) Material of intermediate quality which may or may not be accepted.

If a chance of rejection and acceptance can be agreed to represent a practical interpretation of the word "rare," such as 1 in 20, then the zones (i), (ii) and (iii) referred to can be defined more explicitly.

have a test value less than a defined value, the number of individuals in the sample being 5 per cent of the bulk, provided not less than 10 articles be tested."

The sampling clause permits any number of individuals greater than 9 in the sample, and the permissible number of individuals in the sample which can have a test value below the specification value changes discontinuously, thus :—

From 10 articles	2 can be below specification value.
From 11 to 20 articles	3 can be below specification value.
From 21 to 30 articles	4 can be below specification value.

and so on.

The dots shown in the figure indicate, for each size of sample, the proportion of articles below the specification value which can be present in bulk supplies for a 1 in 20 chance of acceptance and rejection, respectively. The upper set of dots represents the consumer's safe level, and the lower set the manufacturer's safe level. For example, consider samples of 15. If 44 per cent of the articles in the bulk were defective, then only once in 20 selections

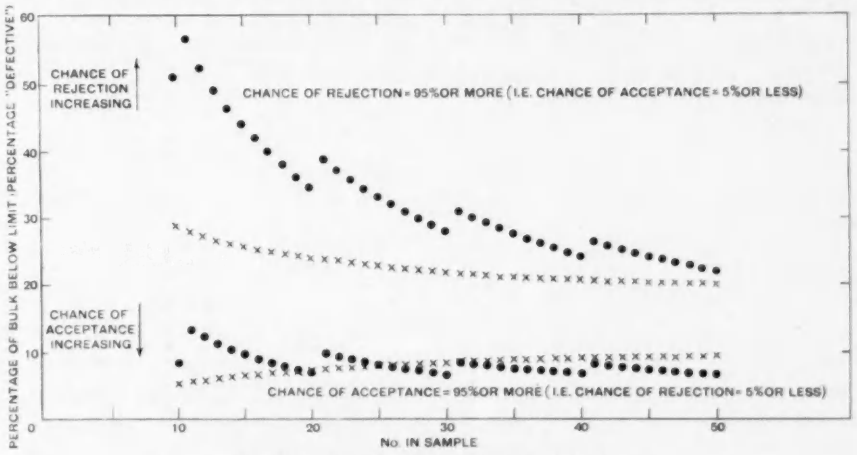
PERCENTAGE OF BULK BELOW LIMIT (PERCENTAGE "DEFECTIVE")
CHANCE OF REJECTION INCREASING
CHANCE OF REJECTION = 95% OR MORE (I.E. CHANCE OF ACCEPTANCE = 5% OR LESS)
CHANCE OF ACCEPTANCE INCREASING
CHANCE OF ACCEPTANCE = 95% OR MORE (I.E. CHANCE OF REJECTION = 5% OR LESS)
NO. IN SAMPLE
0
10
20
30
40
50
60

Fig. 21.—Implications of a specification.

• • Clauses based on percentage "defective" in sample.

× × Clauses based on average of sample.

The level of quality for a bulk supply for which there is a 1 in 20 chance of its being rejected may be described as "the manufacturer's safe level," and the level of quality for which there is a 1 in 20 chance of its being accepted may be described as "the consumer's safe level."

The region of product quality between these two levels is a region of uncertainty, and its range is a practical indication of the efficiency of the specification.

To illustrate the foregoing discussion fig. 21 has been prepared based on the following specification criterion :—"That not more than 10 per cent plus one article, fractions counting as whole numbers, shall

would the product be accepted, because not more than 3 articles in the sample were defective. On the other hand, if 10 per cent of the articles in the bulk were defective then only once in 20 selections would the product be rejected, because more than 3 articles in the sample were defective.

It is clear that this type of specification clause maintains, approximately constant, the chance of rejection of material having 8 per cent below the specification value, whatever the size of the sample. However, it will be noted that, when the size of sample is small, the proportion of "defectives" in the bulk can be quite large before the chance of acceptance is reduced to as low as 1 in 20, but the distance between the two

sets of dots rapidly diminishes as the size of the sample increases.

The clause already considered involves no knowledge of the mean value and spread in the quality of the product. However, suppose it were known that for this product the distribution can be considered as Gaussian; that the spread is closely constant; but that the average value may change. The change in the average value will then determine the proportion in the bulk supply below a fixed level and, hence, the specification can be drawn up in terms of the average value of the sample. (This state of affairs is not infrequently met in practice.)

In all cases, when specification clauses are under consideration, an investigation on the lines of the foregoing is essential, if their real implications to both manufacturer and consumer are to be appreciated.

Finally, close study of the problems arising in the course of drawing up intelligent specifications will emphasize the very considerable advantages to be gained by the adoption of a system under which the manufacturer guarantees to maintain his product at an agreed standard, his guarantee being founded on evidence, available to an independent standardizing authority, that efficient control of processes and products is continuously maintained.

TABLE 2.

TEST RESULTS FOR PRODUCT OF MULTI-HEAD MACHINE.

Side of machine	**Left**						**Right**					
Head number	1	2	3	4	5	6	7	8	9	10	11	12
Individual values	31	28	25	30	23	31	33	34	35	27	37	31
	33	36	30	30	28	27	36	29	35	36	29	39
	29	35	28	29	31	28	23	31	29	30	32	45
	31	29	31	33	32	32	43	39	35	23	42	36
	36	31	31	33	27	28	41	35	32	37	41	36
	34	39	29	37	31	31	33	34	35	39	32	39
	31	35	31	36	35	37	23	31	31	53	33	36
	24	33	36	45	23	43	33	27	39	38	37	51
		27	34	34		46	36	28	36	34	34	32
			38				25	34	27	39	37	
			35				32	36	39	35	36	
							37	34	32	29	41	
							39	33		43	32	
										38		
Head means	31·0	32·6	31·6	34·1	28·7	33·7	33·4	32·7	33·7	35·8	35·6	38·3
Side means	32·0						34·8					
Grand mean	33·6											

Under the conditions here formulated, a limit value can be specified below which the average value of the sample should not fall, from which it is possible to define the manufacturer's and consumer's safe levels. Such levels appropriate to a clause which does not appreciably change the specification from the manufacturer's point of view, are indicated in the figure by crosses. It will be noted that the discontinuous nature of the quality levels disappears, and that the difference between them is considerably reduced, indicating that the efficiency of the specification is improved.

(5) ANALYSIS OF VARIANCE.

As a result of using control charts, coupled with careful consideration of the underlying principles involved when interpreting them, it will be realized that the process is basically one in which variability arising from different sources is compared. The need will be felt for a technique by which total variability may be apportioned to different sources in a manufacturing procedure, and assistance provided for estimating the relative importance of the contributions from such sources. It is at this stage that the engineer will appreciate the utility of the elegant statistical pro-

cedure known as the "Analysis of Variance" which was developed by Fisher.[14, 15]

The square of the standard deviation, *i.e.*, the mean-squared deviation, is known as the "variance," and has the property that contributions to it from various independent sources are arithmetically additive. In the analysis the total variance of a set of data is divided into amounts assignable to specific and chance causes. Statistical theory provides a means for estimating on a probability scale whether the variability attributed to a specific cause arises solely from chance causes or not. The following example of the analysis of variance is described in some detail because it illustrates a very important general principle which, in the opinion of the authors, can scarcely be over-emphasized.

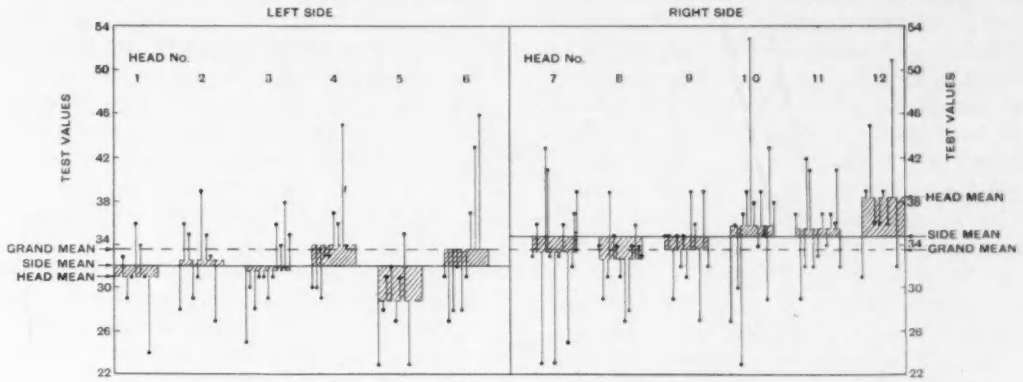

Fig. 22.—Test results for product of multi-head machine.

The data given in Table 2 and illustrated in fig. 22 are the test results obtained some time ago on the product of a machine making articles on 12 different "heads." An inspection, by the engineer in charge, of the results from the 12 heads naturally suggested that head No. 5 was producing poor work and needed head No. 5. So far, the assumption is that the data for all 12 heads are subject to the same system of chance causes. The analysis would indicate that No. 5 alone is affected by a cause other than the general system of chance causes. However, knowledge of the construction of the machine throws suspicion on the validity of this assumption and conclusion, because an essential portion of the driving mechanism consists of two parts, one part being common to the six heads constituting what may be conveniently described as the left side of the machine, and the other part common to the six heads constituting the right side. The engineer might be expected to take the next step of calculating the mean results for the left and right sides respectively; to note that they

TABLE 3.

ANALYSIS OF VARIANCE.

Source of variation	Sums of squares of deviations	Degrees of freedom	Mean variance	Z	Significant levels of Z	
					1 in 20	1 in 100
(*a*) Difference within heads	3,139	116	27·1			
(*b*) Differences between heads within sides ..	392	10	39·2	0·18	0·32	0·45
(*c*) Differences between sides	233	1	233	1·07	0·68	0·97
Total	3,764	127				

Note: For (*b*), $Z = \frac{1}{2}\log_e \frac{(39{\cdot}2)}{(27{\cdot}1)}$

For (*c*), $Z = \frac{1}{2}\log_e \frac{(233)}{(27{\cdot}1)}$

differ, and that head No. 5 is associated with the lower "side" mean.

The questions will arise: Does the difference between the two side means indicate a fault in the associated mechanisms, and does head No. 5 need attention?

A formal statistical analysis following the technique of the analysis of variance will give sound guidance and confidence in answering these questions. In Table 3 the total variability of the 128 results has been divided into three parts (*a*) the amount of variability which is associated with the product of any one head, (*b*) the variability which is exhibited between the products of the heads on the same side of the machine, and (*c*) the variability which is contributed by the two different halves of the machine.

The variability (*a*) is that attributed to the system of chance causes, and (*b*) and (*c*) may be described as the variabilities due to assignable causes, because there are technical reasons for attributing them to particular portions of the mechanism. The chance variation would give rise to some variation in (*b*) and (*c*), even if the different parts of the machine had no influence. The magnitude of the variabilities (*b*) and (*c*) can be compared after allowance has been made for the contribution of the chance causes (*a*).

The procedure is summarized in Table 3. Here are shown the values of a quantity usually denoted by Z, which are calculated from the variabilities to be compared. In addition, from published data,[14] values of Z are given which might be expected only 1 in 20 times and 1 in 100 times if the results for the separate heads were random samples from bulk supplies subjected only to the operation of the chance causes.

The relatively high value of Z (1·07) for the "between sides" contribution indicates that this is significant, whereas the low value (0·18) for the "between heads, within sides" contribution indicates that this may have arisen from chance causes. In other words, the level of quality of the product is different for the two sides of the machine, but, on any one side, the six heads are producing work of substantially the same quality. There is no longer strong evidence that head No. 5 is producing work inferior to the other heads with which it is associated. The practical result of this is that something common to all the heads on the left side and not associated with those on the right side should first receive attention, and the individual heads not be disturbed. Actually, as a result of correct analysis, a fault was discovered in the mechanism associated with the left side, and, after correction, the quality of the product of the whole machine rose to that of the right side. No attention was given to head No. 5.

Without such a formal analysis based on statistical principles any technical man would almost certainly have given attention to head No. 5, and, unfortunately, because the low value for the product of head No. 5 was due to chance, it is highly probable that later data would have convinced him that attention had had the desired effect. On the other hand, without the knowledge of the operation of the machine a statistician could not have been expected to have gone further than the first stage of the analysis in which head No. 5 compared unfavourably with the other eleven heads.

In practice more complicated problems arise, but after experience with some simple examples the ability to apply the principles underlying this procedure above referred to will be acquired and its value properly appreciated.

(6) FURTHER EXTENSIONS OF STATISTICAL TECHNIQUE.

The principles which form the foundation of the various techniques illustrated in the foregoing Sections are capable of considerable extension.

One of the most important is probably that in which it is desired to know whether or not different measured characteristics of a product are essentially related. The existence of any such relation can often be used to avoid destructive testing at various stages of the manufacture of a product, offering considerable economies in the cost of maintaining effective quality control.

Further, it is also necessary to make due allowance for any relation between observed characteristics in order to draw correct conclusions from the study of data.

Examples of the use of statistical technique in more complicated problems are contained in various papers of the Supplement to the *Journal of the Royal Statistical Society*. Many of the most valuable are based on data derived in the course of agricultural research, but a study of these is unlikely to be useful without considerable experience with simpler problems of the type described in previous Sections.

(7) GENERAL.

The foregoing review cannot, by reason of limitation of space and time, do more than provide an introduction to general principles and indicates a line of development that can be followed when applying probability theory to the everyday problems arising in a manufacturing industry.

Throughout the treatment it will be realized that fundamentally the assumption first made is that the variability exhibited by the product sampled arises entirely from a system of chance causes. The test data derived from sample groups of individuals chosen with a knowledge of the manufacturing procedure so that each sample is likely to be equally subject to the system of chance causes are then examined in the light of the primary assumption, using the theory of probability as a guide. Lack of

statistical consistency in the results is an indication that variability, external to the system of chance causes, exists.

Technical knowledge of the manufacturing processes has then to be used to suggest the origin of this variability, and recourse subsequently made to statistical technique to confirm or refute the suggestion. This is the fundamental pattern on which the technique is built.

In the Introduction attention has already been drawn to the advantages which will accrue to the engineer who takes the trouble to acquire the technique. It is fitting to conclude by drawing attention to two important principles which must not be overlooked. The successful operation of a manufacturing procedure demands considerable experience and detailed knowledge of machines and processes. If this experience is associated with specialized knowledge of physics and chemistry few will dispute, at the present time, that the engineer so equipped is better able to fulfil his function in industry than one depending solely on engineering knowledge and experience. Similarly, if to these accomplishments the engineer can add a working familiarity with the principles of statistics he will still further enhance his value to industry. The use of the statistical technique alone cannot solve problems demanding a knowledge of physics, chemistry, and engineering, but it can serve as a powerful aid when drawing conclusions from technical data.

An example has been given where the experienced engineer lacking statistical knowledge, and the statistician lacking practical knowledge, would probably have come to the same erroneous conclusion. Correct diagnosis of the difficulty in the factory was only possible by a combination of the attributes of the two. The necessity for this combination cannot be overemphasized.

Finally, a warning should be sounded against any inclination to believe that acquirement of the technique so inadequately described in this paper can be achieved by listening to papers on the subject or by reading text-books. No one has become a good pianist merely by reading Tobias Matthay's book "Muscular Relaxation Studies," but many have done so by combining concentration of mind with practice of the technique there described. The practice of statistical methods, associated with a keen appreciation of the principles involved, will alone give the engineer command of this valuable industrial tool.

The authors would like to take this opportunity of thanking Professor E. S. Pearson and Dr. B. L. Welch of the Department of Statistics of University College, London, for many opportunities to discuss problems of mutual interest during recent years.

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Fluid Filling-Media for Electrical Apparatus.

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This paper was originally chosen for reading before The Institution of Electrical Engineers in November, 1939. The author discusses the common fluid filling-media for electrical apparatus, particularly transformers, and advances suggestions as to the improvement of British Standard Specification No. 148—1933 ("Insulating Oils for Electrical Purposes"). In addition, the properties of chlorinated diphenyls are discussed and also the particular requirements of oils suitable for condensers. For the sake of clarity, the paper is divided into three headings :—

(1) *Transformer and switch oils.*
(2) *Chlorinated compounds.*
(3) *Condenser oils.*

(1) TRANSFORMER AND SWITCH OILS.

General.

WHEN oil was first adopted for filling transformers and switchgear, use was naturally made of the most suitable existing kind of oil. Even 30 years ago, full approval was accorded to oils that would be instantly rejected to-day, not only with regard to their electrical properties but also to their physical characteristics. With the advent of the first British Standard Specification for insulating oil, however, in 1923, standard methods of test were evolved and specification limits laid down which have gradually led to the improvement of electrical oils, so that in 1927, and again in 1933, British Standard Specification No. 148 was able to be revised and tightened up, with the result that oil manufacturers continued to improve their product until, to-day, electrical insulating oil to British Standard Specification No. 148—1933 could be safely used medicinally, so far has the degree of refining been carried.

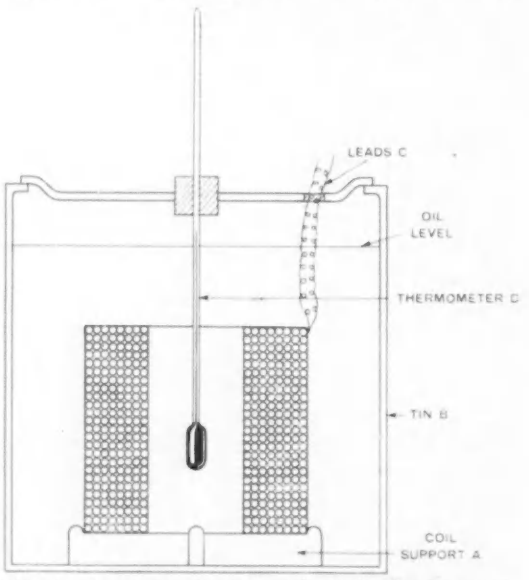

Fig. 1.—Apparatus for the study of the development of acidity in oils.

In some ways, perhaps unfortunately, the classification of oils was adopted on a basis of sludge values taken by a method far removed from service conditions. The cry of the transformer manufacturers then was for as low sludge values as possible, and the criterion of a good transformer oil became a low sludge value, without very much attention being paid to its other properties. This attitude has led oil refiners to develop insulating oils mainly in one direction, and it has become evident that, through curing the development of sludge in an oil by a high degree of refinement, transformer oils of to-day develop acidity instead. The Michie test for sludge, therefore, instead of being used to eradicate unsuitable oils, became the basis of comparison for oils in that the lowest sludge-value oil was adjudged the most suitable. What the Michie sludge test for oil determines as to its suitability for use in a transformer, except as a proof test, is difficult to see, as it does not measure anything in relation to service conditions and is not a true ageing test by acceleration. It is a well-known fact that the oxidation products of an oil at 150° C. are far different from those taken at 90° C., both in quantity and in character. The Michie test has outgrown its original use as a test for the eradication of unsuitable oils.

It is safe to say that 50 per cent of the transformer oil used to-day is Grade "A" oil, typical samples of which give practically zero sludge values. For switchgear purposes, where sludge is of small importance, highly refined oil is quite unnecessary. It follows also that oil would be cheaper if less refined, a point of importance considering the large bulk of oil used. Class "B" oils, which years ago were just attainable with sludge values between 0·7 and 0·8, are nowadays readily procurable at the low figure of 0·3 and less. It is the author's contention that in bringing the sludge values of oils down to these low limits, the danger of development of acidity of oils is now as menacing as the old

danger of high sludge values. It may well be that in the light of modern experience with transformer oils, the highly refined "A" oils of practically zero sludge value employed so widely to-day will be discarded and an oil of a moderate degree of refinement but low developed acidity take its place as standard transformer oil.

Experience and Tests.

It has been the author's experience from time to time to open up transformers which have been in service, and it has been noticed that the underside of the lid has been corroded, the amount of corrosion being far in excess of what one would expect through ordinary rusting. An analysis of this corrosive deposit has revealed the presence of acetic acid due to the development of volatile acidity by the oil. In addition to this, the acid value of the oil itself has been far in excess of its original value, reaching as high in some cases as 4—5 milligrams of KOH per gramme of oil as against an original acid value of approximately 0·05 milligram KOH. The following is an account of an investigation undertaken to obtain some idea of :—

(*a*) The rate of development of acidity under a given set of conditions, approximating to ordinary service transformer duty.

(*b*) The effect on acid development of varnished and unvarnished coils.

(*c*) The difference in development of acidity between oils supplied by different manufacturers.

(*d*) The difference in development of acidity between grade "A" and grade "B" oils.

The details of the test apparatus are shown in fig. 1. Coils were made up as follows, using 20 S.W.G., D.C.C. wire :—

Internal diameter	2ins.
External diameter	3ins.
Width of coil	2ins.

For the test, these were supported on bakelized-paper supports, A, inside ordinary sheet-iron tins of approximately 1½ gallons capacity, B, fitted with lever type lids. The two ends of the coil were taken through small holes in the lid of the container, C, and in each case a thermometer, D, was fitted through the lid so as to have its bulb centrally disposed with reference to the coil. For four of the oil samples (two grade "A" and two grade "B") two tins were prepared, one tin containing a standard coil untreated except for drying for 18 hours at 100° C., and the other containing a standard coil, varnish-impregnated with a yellow-oil-base insulating varnish. For the rest of the oil samples, impregnated coils only were used.

The coils were connected to an A.C. supply such as to give a steady condition of temperature of 85°—90° C. as read on each thermometer, and the test was carried on continuously for 6 months (except for an occasional week-end shutdown). The acid value of the oil in each tin was taken at the end of each month, the samples for test being extracted through the thermometer holes. These results are set out in Table 1. At the end of 6 months each tin was opened and the inside conditions were noted (see Table 2). In addition, the average developed acidity for the grade "A" oils against the grade "B" oils is shown in fig. 2.

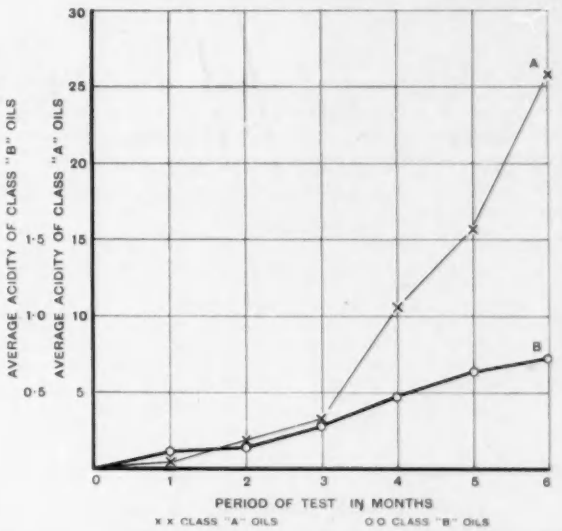

Fig. 2.—Curves showing average development of acidity of Class "A" and "B" oils over a test period of 6 months. Acidity figures in milligrams KOH per gramme of oil.

Conclusions Drawn from Investigation.

Taking general tendencies as given by this investigation, the answers to the points to be determined are :—

(*a*) For the grade "A" oils, there seems to be a definite acceleration in formation of acid during the third month, and even after six months there is no evident tendency to decrease the rate ; in fact, the average curve indicates an increasing rate of acid formation at six months.

With regard to the grade "B" oils, the rate of rise over the six months is fairly uniform, with definite indications that the rate of formation of acid is decreasing after approximately five months.

(*b*) The "A" oils containing varnished coils developed slightly more acid than the corresponding oils containing unvarnished coils. An examination of Table 2 shows, however, that the lids of the tins in the former cases were free from rusting, but were rusted in the latter case. This would seem to indicate that more volatile acidity was developed in the case of unvarnished coils. The "B" oils containing varnished coils developed less acidity than the corresponding oils containing unvarnished coils and, judging by the state of the tin lids, very little or no volatile acidity was developed at all.

(*c*) Generally speaking, there was little difference shown between the various grade "A" oils, and although more relative difference was shown between the various "B" grade oils the worst measured acidity developed was, in actual terms, small in comparison with the grade "A" results.

(*d*) The difference in developed acidity between the "A" and "B" oils is very marked, with the less acidity in decided favour of the grade "B."

This acid value corresponds to values obtained in the test previously described after 3—4 months (with unvarnished coils) and would, therefore, seem a suitable test which could easily be included in a specification. The next logical step is, that if a grade "B" oil of sludge 0·2—0·3 develops less acidity than a grade "A" oil of practically zero sludge value, it is logical to assume that, using the same base oil but of a less degree of refinement, say with a sludge

TABLE 1A.

Acidity Development of Grade "A" Oils (in milligrams KOH per gramme of oil).

Sample	Varnished coils						Unvarnished coils					
Ref.	1 month	2 months	3 months	4 months	5 months	6 months	1 month	2 months	3 months	4 months	5 months	6 months
A	0.028	0.11	4.09	11.0	16.6	27.7	0.14	0.182	5.0	8.75	12.3	16.0
B	0.028	0.42	5.01	12.2	17.5	26.8	0.02	0.42	5.49	7.4	16.4	22.3
C	0.02	0.07	1.90	9.1	14.3	23.7						
D	0.014	0.07	1.81	9.5	14.8	25.5						
Average	0.023	0.17	2.6	10.5	15.8	25.9						

TABLE 1B.

Acidity Development of Grade "B" Oils (in milligrams KOH per gramme of oil).

Sample	Varnished coils						Unvarnished coils					
Ref.	1 month	2 months	3 months	4 months	5 months	6 months	1 month	2 months	3 months	4 months	5 months	6 months
A	0.028	0.07	0.37	0.39	0.58	0.55	0.063	0.435	1.39	2.38	2.52	3.2
B	0.042	0.168	0.42	0.85	1.25	1.4	0.05	0.084	5.6	9.4	10.2	12.6
C	0.112	0.12	0.14	0.16	0.196	0.28						
D	0.028	0.08	0.42	0.83	0.86	0.91						
E	0.169	0.18	0.196	0.25	0.33	0.40						
Average	0.076	0.124	0.309	0.49	0.633	0.71						

Proposals.

Taking into consideration this investigation and also the sum total of the experience of transformer-oil users, the author considers that a strong case has been made for the adoption of an oil of the lowest acidity development, even at the expense of the low sludge values now obtained. The next point to arise, however, is this: How is the development of acidity to be measured? Seeing that the Michie sludge test is the standard British "ageing" method for oils, the logical proposal would seem to be to carry it a step further and use it for measuring developed acidity as well as sludge.

It has been found that if, after separating out the sludge, the acid value of the remaining liquor be determined, the normal class "B" oil gives a value of 3—4 milligrams of KOH per gramme of oil. of 0·4—0·6, the latter oil ought to develop still less acidity. In conjunction with a well-known oil supplier, this supposition has been checked with the excellent result that an oil was produced with an acidity developed figure, taken after the standard Michie sludge test of 0·96 milligram KOH and a sludge value of 0·46 per cent. This sludge value is well within the maximum British Standard Specification No. 148—1933 figure of 0·8 per cent, and such an oil should not develop an undue amount of sludge under the severest service conditions, nor should it develop much acidity in service.

The way in which the degree of refining influences the character of the oxidation products may be explained by the resins in the partially refined oil being oxidized to a solid substance, *i.e.*, sludge, and that these products in some way prevent the oxidation

TABLE 2.

Condition of Oils after Six Months.

Sample Ref.	Grade	V—coil varnished UV—coil unvarnished	Electric strength of oil breakdown voltage/ (seconds for breakdown)	Colour of oil	Amount of deposit in oil	Colour of deposit in oil	Deposit on sides of tin	Deposit on lid of tin	Amount of deposit on coil	Colour of deposit on coil	Nature of deposit on coil	Moisture in oil* (by 'crackle' test)
A	A	UV	21,000 volts 28,000 volts 25,000 volts Instantaneously in each case	Very dark brown	Copious	Dark brown	Slight amount, dark brown, sticky	Large amount, dark brown, rusty and gummy	Copious	Dark brown	Slimy	III
A	A	V	30,000/2 30,000/5 30,000/5	Light yellow	None	—	Very slight amount light yellow, sticky	None	Copious	Dark greenish brown	Sticky	II
B	A	UV	30,000/40 30,000/20 30,000/25	Dark brown	Slightly clouded	Whitish	Slight amount, yellow, sticky	Slightly rusted	Copious	Greenish brown	Soft	II
B	A	V	30,000/6 30,000/5 30,000/11	Light yellow	None	—	Slight amount yellow, sticky	None	Copious	Dark green	Slimy	II
C	A	V	30,000/60 no breakdown. Breakdown at 35,000	Light yellow	None	—	Slight amount, white, soft	None	Slight	Dark brown green	Soft gritty	I
D	A	V	30,000/60 no breakdown. Breakdown at 4,000	Light yellow	None	—	Slight amount, colourless, soft	None	Slight amount in blobs	Light greenish brown	Sticky	I
A	B	UV	30,000/20 30,000/10 30,000/25	Light yellow	None	—	Very slight amount, white, sticky	Very slight rusting	Slight	Dark brown greenish	Soft	II
A	B	V	16,000 volts 12,000 volts 14,000 volts Instantaneously	Dark brown	Fairly copious	Dark brown	Slight amount, dark brown	None	Copious	Dark brown	Soft	II
B	B	UV	30,000/8 30,000/4 30,000/6	Dark brown	Slight	—	Fairly thick, reddish brown, sticky	Slight rusting	Copious	White	Sticky	II
B	B	V	20,000 volts 25,000 volts 22,000 volts Instantaneously	Dark brown	Slight	—	Fairly thick, dark brown, soft	None	Copious	Dark brown	Soft	II
C	B	V	30,000/5 27,000 30,000/11	Light reddish brown	Slight	Light yellow	Yellow, soft	None	Slight	Yellow	Soft	I
D	B	V	30,000/2 30,000/5 30,000/5	Light brown	Slight	—	Slight amount dark brown, sticky	None	Very slight	Light yellow	Soft	II
E	B	V	15,000 volts 11,000 volts 12,000 volts	Light brown	Slight	—	Slight amount, light brown, soft	None	None	None	—	II

* I—None. II—Slight trace. III—Trace.

of the hydrocarbon portions. In the case of highly refined oils there is no protection of the hydrocarbon portions by polymerization products, it being assumed that direct oxidation of the hydrocarbons produces acidity.

It is also logical to assume that the composition of the actual crude oil itself plays a large part in the determination of the properties of the refined oil.

The method of refining is also said to have a large bearing on the characteristics of the refined oil. It is considered in some quarters that methods of refining which remove all the aromatic unsaturated molecules produce insulating oils which easily form acids in service, whereas by removing all aliphatic constituents an oil of low acidity formation is produced.

It thus appears that the following factors all influence to some degree the formation of acid in an oil under service conditions :—

(1) Origin of crude oil and its characteristics.
(2) Method of treatment of crude oil to produce basic transformer-oil distillate.
(3) Method of refining blended basic oil distillate.
(4) Degree of refining.

The investigation of these variables provides much scope for the enterprising oil supplier.

Suggestions for Test and Specification Purposes.

In the treatment of acidity development of oil in this article so far, little mention has been made of volatile acidity, the results of which are usually shown more plainly in transformers than residual acidity in the body of the oil, as it produces corrosion of metal parts above oil level. The Michie sludge test would tend to carry over volatile acidity unless condensed in a special trap, so that the measurement of the acidity of the sludge liquor does not necessarily tell a complete story. So far as the limited experience of the author on this particular point is concerned it appears reasonable to assume that if steps are taken to keep down residual developed acidity, then volatile acidity will also be reduced. In any case, the provision of breathers on transformers has been found to reduce corrosion through volatile acidity, although it is not known if this step prevents its formation or merely gets rid of it before it has time to do damage.

British Standard Specification No. 148—1933 is considered inadequate on two points, therefore, in that :—

(*a*) The sludge test bears no relation to service conditions unless a relationship between deposit at 150° C. and, say, 110° C., is established.

(*b*) No mention is made of developed acidity after ageing.

The remedy for the former is either to determine the relationship mentioned, substitute a sludge test at 110° C. for a longer period, or substitute a sludge test at 110° C. for the same period and reduce the specification limit. The remedy for the latter is, of course, to measure and specify a limit for the developed acidity after the sludge test.

The following method has been found efficacious for the determination of developed acidity after sludging :—

> Filter off the sludge and make up the filtrate by addition of petroleum ether to 500 cm.^{3} Take 200 cm.^{3} of this, add 100 cm.^{3} of neutral alcohol with 2 cm.^{3} of alkali blue indicator, and allow to stand for 24 hours. Titrate with N/10 KOH.

Using this method, the author considers it feasible to fix a maximum developed acidity of 1·5 milligrams KOH.

(2) CHLORINATED COMPOUNDS.

Introduction.

The greatest drawback of oil as a filling medium is its inflammability, and in consequence of such disastrous switchgear and transformer fires as those at Bradford and Kingston, and the danger of fires caused by incendiary bombs, the thoughts of engineers are strongly turned towards the only non-inflammable liquids at present commercially available, *i.e.*, synthetic chlorinated compounds. The commonest of these are the chlorinated diphenyls made in America and on the Continent, but not so far manufactured in this country.

Chlorinated Naphthalenes.

The drawback to the use of these for filling electrical apparatus is that there is considerable doubt as to their stability under heat, in the liquid form. The author has found that the liquid variety tend to evolve chlorine at temperatures of approximately 100°—110° C. and on this account little use has been made of them, except possibly for liquid fuses. In the solid form, better known as the Seekay Waxes, they have been used in the electrical industry, to a limited extent, as a non-inflammable filling compound in place of bitumen and other waxes.

Chlorinated Diphenyls.

It is a well-known fact that there have been marketed in America for some years now, types of transformers filled with non-inflammable liquids known as pyranols, the composition of which is largely chlorinated diphenyl. On the Continent also there are the clophens, which, too, are largely composed of chlorinated diphenyls. The degree of inflammability of pure chlorinated diphenyl, more familiarly known in this country as Aroclor, depends on the degree of chlorination. For instance, a compound 48 per cent chlorinated has a flash-point of 196° C. but is self-extinguishing. A compound 54 per cent chlorinated has no flash-point and, in addition to being itself non-inflammable, as is shown

later in this article, acts in the manner of a fire-extinguishing medium. The 54 per cent chlorinated compound Aroclor, which is largely pentachlor diphenyl, has two great drawbacks, however. The first is its price in this country, which is approximately 6 times that of ordinary transformer oil. Its use is thus precluded in electrical apparatus unless buyers of gear are prepared to overlook cost in the interest of safety of stations from fire risk, and it may well be that in face of the several serious electrical fires which have occurred during the last few years, the use of chlorinated diphenyls in some form or another may become, if not standard practice, at least a standard type of product of the leading manufacturers of electrical gear.

Fig. 3.—Experiment to determine the non-inflammability of 54 per cent Aroclor. Tank 1 contains blazing switch oil, tank 2 is an open drum of Aroclor.

The second serious drawback is the shape of the viscosity/temperature curve. At 20° C. the 54 per cent chlorinated diphenyl is a pale yellow, rather thick, oil; at 60° C. it is practically as fluid as transformer oil; at +5° C. however, it is practically solid. This compound is thus quite unsuitable for use in transformers unless thinned by the admixture of such other chlorinated synthetics as trichlorbenzene. This article is confined, however, to pure chlorinated diphenyl compounds, so that the only apparent use of the 54 per cent chlorinated compound is as a non-inflammable filling medium for the busbar and current-transformer chambers of switchgear. The following interesting experiment has been carried out to determine the extent to which 54 per cent chlorinated Aroclor is non-inflammable. This experiment is illustrated by the two photographs shown in figs. 3 and 4, the first of which shows an open welded tank containing switch oil which was fired by heating from below with a fierce conflagration as depicted, actually obtained by burning wood wool. This steel tank is marked (1) in the photograph. Beside this steel tank containing the blazing oil is a small steel drum, marked (2), which contained 54 per cent chlorinated Aroclor, this drum having an open bung at the top. It will be seen that whereas the oil is burning freely, the Aroclor is not burning at all.

Fig. 4 shows that when the Aroclor in the small drum had become heated so as to have become quite fluid (approximately 150° C.), it was removed and by means of a portable pump it was sprayed on to the burning oil. This photograph shows the lance and jet of the spraying apparatus, and also shows that the Aroclor has extinguished the oil fire, the flame and fumes from the burning oil being replaced by the dense white cloud of heavy fumes from the Aroclor.

This experiment demonstrated not only the non-inflammability of the Aroclor but that, in addition, it has the valuable property of acting as a fire extinguisher.

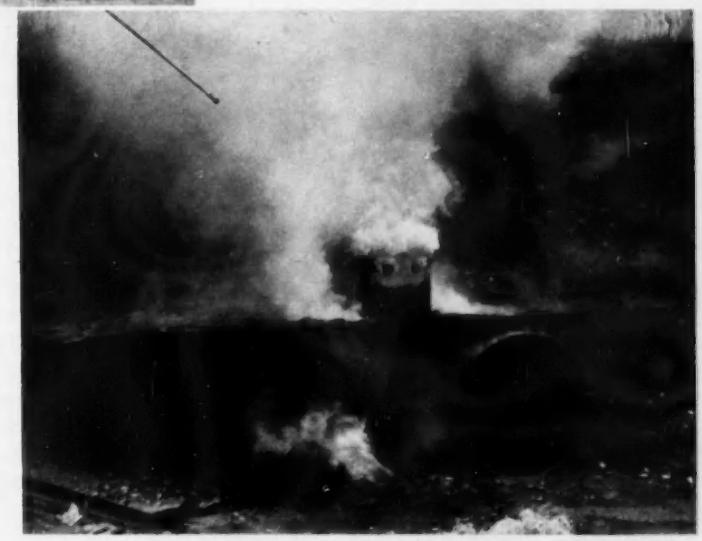

Fig. 4.—Second stage of the experiment illustrated in fig. 3. The Aroclor, heated to 150 degs. C., sprayed on to the burning oil, has extinguished it.

For use as a non-inflammable filling medium for transformers in place of transformer oil, the maximum chlorination compatible with suitable viscosity is 48 per cent. Here again, the shape of the viscosity/temperature curve is very steep compared with that for transformer oil, but the 48 per cent chlorinated Aroclor is just suitable. It is a water-white liquid

at 20° C., and its viscosity at 35° C. is approximately the same as that of an "A" grade oil at 20° C. At 60° C., however, it is much thinner than "A" grade oil, but at 0° C., while A30 grade transformer oil is perfectly fluid, this Aroclor is quite viscous, its actual cold point being – 9° C. Thus at high ambient temperatures 40 per cent chlorinated Aroclor would possess an advantage over transformer oil as regards cooling by convection, but at 20° C. this advantage would be reversed, and at temperatures approaching zero Centigrade the Aroclor would be definitely dangerous. Some of the other properties of 48 per cent chlorinated Aroclor are given in Table 3.

TABLE 3.

Specific gravity at 20° C.		1.456
Coefficient of expansion between 20° C. and 65° C.		0.000702
Acidity, as supplied		0.01 milligram KOH
Sludge value, taken as for transformer oil British Standard Specification No. 148—1933		Nil
Acidity after sludge		0.05 milligram KOH
Thermal conductivity at	30° C.	0.0613 B.Th.U./hr./sq. ft./deg F./ft.
	60° C.	0.0698
	100° C.	0.0800
Specific heat at	30° C.	0.290
	60° C.	0.339
	100° C.	0.405
Action on metals, insulating materials, etc.		Tests carried on over a period of a year at 100° C. have shown that this Arcoclor has deleterious action on copper, brass, aluminium, iron, and such insulating materials as paper, empire cloth, and cotton tape.

It will be noted that the above grade of Aroclor has no sludge value as determined by the standard method laid down for oils, and, in addition, that there is practically no acidity developed during the sludge test. This in itself is a good indication of the stability of the material at the sludge temperature of 150° C.

It will be noted that the specific gravities of these chlorinated compounds are well in excess of unity, which imposes a condition upon the designer that no bare conductors should be brought through the surface of the medium in case water lying on the top should cause a breakdown along the surface of the medium. No suitable non-inflammable medium has so far been found for use in switch tanks or other places where arcs or sparking occur normally. The chlorinated compounds decompose under an arc and evolve free hydrochloric acid and probably free chlorine, and although it is possible for these to be absorbed above the level of the medium a risk is involved which few would be prepared to countenance.

An interesting property of Aroclor is the high dielectric constant, which is approximately 5 as compared with 2 for transformer oil. This fact is made use of in the manufacture of power-factor correction condensers, where a 50 per cent increase in capacitance is obtained through the use of Aroclor instead of a mineral oil. Here again, however, the high price of Aroclor prevents its wider use in this field.

(3) CONDENSER OILS.

Introduction.

Although oils used in the manufacture of power-factor-correction condensers are in the main pure mineral hydrocarbon oils, service conditions are so severe that they cannot be used as delivered by the oil refiners as can safely be done for transformers. Condenser oils have to be specially treated owing to the fact that they are not only used for filling condensers but also for vacuum-impregnating the paper dielectric. The following gives some idea of the special properties required of condenser oils and the special treatments attending their use.

Oil Impregnation of Condensers.

The commonest type of power-factor-correction condenser is the paper-dielectric type, where the oil is used to add electric strength to the dielectric and, in addition, to fill the box containing the condenser spools, so as to exclude air and to aid cooling. It is of great importance that this type of condenser be hermetically sealed and that the vacuum impregnation be so efficient as to exclude all air voids in the dielectric. The stress in condensers is very high compared with other forms of electrical apparatus and, in the event of voids being formed, ionization occurs which causes decomposition of the oil with subsequent breakdown. A further phenomenon in oil-filled condensers is that certain oils themselves are affected by severe electric stress with the evolution of gas, mainly hydrogen. This occurs mostly at the edge of the foil electrodes of the condenser where the stress is highest. Such oils must, therefore, be avoided. There appears to be a definite relationship between the amount of saturation of an oil and its tendency to generate or absorb gas, but this phase of the subject is as yet not fully understood. The estimation of the amount of saturation of condenser oil is usually made by iodine values. The apparatus shown has been used to study the absorption of gas of oils under electric stress. The body is constructed of soft glass, as shown in fig. 5. A small piece of platinum wire is fused into the base of the chamber containing the oil, and the central electrode, which may be of nickel or tinned copper, rests on this ; the collar at the top of the tube keeps it central. The outer electrode consists of a thin lead sheet which is wrapped round the outside of the glass tube and is simply wired into position. The electrode part of the apparatus is immersed in a beaker of oil for the test, and a potential of approximately 7,000 volts is applied. The oil under test is first saturated with

hydrogen and then sucked into the tube via the manometer. When full, hydrogen is admitted through the three-way cock so as to enclose a suitable volume in the apparatus, and the amount of gas absorbed or propagated can be read off on the manometer, after suitable correction for the difference in the levels. Tests have shown that, under stress, oils either absorb or evolve hydrogen, and it is this property which is related to iodine value. It has been found that those oils having an iodine value of less than

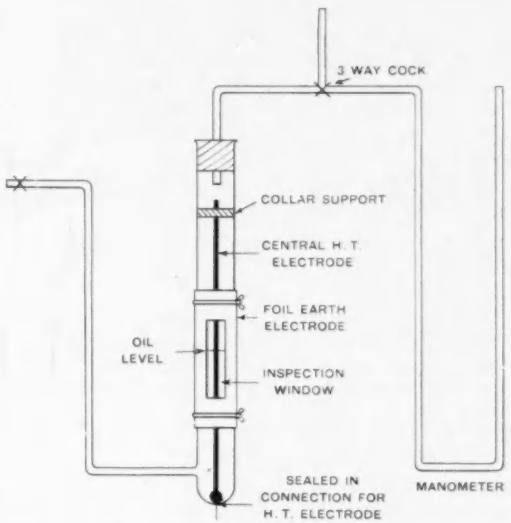

Fig. 5.—Apparatus for studying the absorption of the gas of oils under electric stress.

6·5 give off hydrogen under stress; those with an iodine value exceeding 6·5 absorb hydrogen under stress.

The importance of this in relation to condensers will be appreciated; if through inefficient vacuum treatment the condenser has air voids trapped in the layers of paper or foil, these voids may be increased through the evolution of hydrogen, depending upon the iodine value of the oil. It is in the boundaries of these voids, if they are large enough, that the phenomena of "cheesing" occurs. "Cheesing" is the formation of a wax-like solid well known in cable manufacture as being produced at voids under stress in oil-filled cables.

In other words, an oil having a higher iodine value than 6·5 will tend to counteract the effects of slightly imperfect vacuum impregnation, whereas the use of an oil of iodine value of less than 6·5 will cause the effects of imperfect impregnation to be gradually increased, with unavoidable subsequent failure. Wherever the condenser has been perfectly impregnated the gas-absorbing or gas-propagating property of the oil will not matter, as it is only where voids exist that the correct conditions for evolution of gas under stress obtain.

An oil of higher iodine value than 6·5, however, is relatively unsaturated and is, therefore, more liable to oxidation than a highly refined oil of iodine value of, say, 2. This type of oil is also more easily contaminated and precautions must be taken during its use to ensure that it does not deteriorate through oxidation in the impregnating process.

Precautions to be taken with High-Iodine Value Oil.

As delivered from the oil manufacturers, new condenser oil has a power factor of approximately 0·001 taken at a temperature of 70° C., which temperature is usually accepted as being the maximum permissible for operating condensers in service. It is found that after such oil is passed through two or three impregnating cycles, the power factor has increased owing to the oxidation of the oil. This means that the oil must either be scrapped or re-conditioned in some way so as to restore the power factor to its original value. It has been found efficacious to treat such oil with activated fullers' earth, by which treatment the products of oxidation in the oil are removed, and the oil is clarified and again restored to a usable power-factor value. It has been found sufficient to mix with the oil approximately 2½ per cent of fullers' earth, agitate for an hour and then pass the oil through a stream-line filter. The oil can be re-conditioned in this way many times and, as it is only the excess oil from the impregnating process which requires treatment, it is rarely found necessary to scrap any oil after use.

Precautions to be taken with Low-Iodine Value Oil.

As this type of oil is relatively highly refined, it is much more resistant to oxidizing influences than high-iodine-value oil and is found to deteriorate relatively slowly during use, so that the fullers' earth re-treatment method above described is rarely necessary. Using this oil, however, it is more necessary to attain high vacuum during the impregnating process so as to ensure absolute absence of voids in the condenser spool after treatment.

Where condensers are used with a free oil level, the choice of the most suitable oil becomes very difficult as, on the one hand, if a highly refined oil is used there is more chance of gas evolution, owing to the physical absorption of air by the oil and the subsequent danger of air voids finding their way into the dielectric. On the other hand, if a less highly refined oil is used, there is more danger of oxidation. The majority of power-factor installations in this country, however, provide hermetically-sealed condensers.

In conclusion, the author wishes to express his thanks to W. B. Dick & Co. and the Monsanto Chemical Co. for data supplied for inclusion in this paper.

Technical Literature.

A DIGEST OF RECENT BOOKS, ARTICLES AND PAPERS BY MEMBERS OF THE G.E.C. AND ITS ASSOCIATED COMPANIES.

SOME IMPEDANCE CHARACTERISTICS OF TAPPED RESONANT-LINE CIRCUITS.*

By W. E. Willshaw (Research Laboratories).
Phil. Mag., Vol. 29, p. 572, June, 1940.

The paper discusses some properties of tapped uniform lines relevant to their use as frequency stabilizing elements in short wave oscillators.

THE HEATING OF THIN MOLYBDENUM FILMS IN A HIGH FREQUENCY FIELD.*

By D. A. Wright (The M.O. Valve Co. and the G.E.C. Research Laboratories).
Proc. Phys. Soc., Vol. 52, p. 253, 1940.

Molybdenum films, formed by evaporation of Mo on to a glass substrate, are investigated by measurement of the heating occurring on application of a high frequency electric field. The effect is observed of varying glass temperature during deposition, and of contaminating the glass with oxygen, hydrogen, and with layers of metallic oxides or fluorides. The stability of the films is examined and related to the processes of agglomeration and recrystallisation. The effect of bombarding the films with electrons is also investigated.

THE DIFFUSION OF OXYGEN IN COPPER.*

By C. E. Ransley (Research Laboratories).
J. Inst. Metals, Vol. 6, p. 537, November, 1939.

The rate of diffusion of oxygen in solid copper at different temperatures has been calculated from measurements of the rate of deoxidation of copper strips under certain conditions. The diffusivity constants and activation energy of the migratory process are derived. The mechanism of the embrittlement of copper on heating in reducing atmospheres is discussed on the basis of the new data obtained.

GRAPHICAL METHODS FOR THE DETERMINATION OF DISTORTION IN VALVES WITH RESISTIVE LOADS.*

By A. Bloch (The M.O. Valve Co. and G.E.C. Research Laboratories).
Wireless Engineer, Vol. 16, p. 592, December, 1939.

Simplified graphical methods are shown for the derivation of D.C., fundamental, 2nd, 3rd and 4th harmonic components in the anode current of a valve with sinusoidal grid excitation, if the anode current is known for 5 valves of grid voltage evenly spaced over the range of grid swing. The methods follow directly from a rewriting of the well-known formulae for a "5-point analysis" and form at the same time an easy method for memorizing these formulae.

STATISTICS AND ENGINEERING PRACTICE.*

By B. P. Dudding and W. J. Jennett (Research Laboratories).
Jour. I.E.E., Volume 87, p. 1, July, 1940.

This article is reprinted on p. 195 of this issue of the *G.E.C. Journal*.

THE DEPOSITION OF OXIDE COATING BY CATAPHORESIS.*

By M. Benjamin and A. B. Osborn (The M.O. Valve Co. and G.E.C Research Laboratories).
Proc. Faraday Soc., Vol. 36, p. 287, January, 1940.

Description of experimental work which has led to production of suspensions of alkaline earth carbonates having the advantages of colloidal suspensions. The particles can be deposited cataphoretically although their size may range from ½ micron to 100 microns.

RADIATION PROPERTIES OF DIFFERENT SUBSTANCES WITHIN THE TEMPERATURE RANGE 250° AND 800°C.*

By M. Pirani (Research Laboratories).
J. Scientific Instruments, Vol. 16, p. 372, December, 1939.

A simple method is described of comparing the total radiations from samples at high temperatures. Quartz glass is used as an easily reproducible reference standard. An attempt has been made to obtain approximately absolute figures by calculating the emission of the standard substance from measurements of its transmission for different wavelengths in the infra-red at room temperatures.

AN X-RAY STUDY OF NATURAL AND ARTIFICIAL MULLITES.*

By H. P. Rooksby (Research Laboratories).
J. Soc. Glass Technology, Vol. 23, p. 338, December, 1939.

It is shown by X-ray investigation of mullite that there are three varieties distinguishable by the differences in axial ratios of the unit cell. The results of this investigation are discussed.

DISCOMFORT GLARE IN LIGHTED STREETS.*

By R. G. Hopkinson (Research Laboratories).
Trans. I.E.S., Vol. 5, p. 1, January, 1940.

Glare which affects comfort or causes distraction is distinguished from glare which affects ability to see. The results of an investigation to determine the factors which cause discomfort glare in lighted streets are summarised and the influence of these factors on problems of street lighting design is studied.

IMPEDANCE NETWORKS.*

By R. S. Rivlin (Research Laboratories).
Reports in Progress in Physics, Vol. 6, p. 389, 1939.

A review of the advances which have been made in the theory and design of impedance networks. The review, however, is limited to the behaviour of networks in their steady-state.

VARIABLE AIR CONDENSERS.*

By R. F. Proctor (Research Laboratories).
Wireless Engineer, Vol. 17, p. 257, June, 1940.

A method is described of determining the residual parameters of a variable air condenser, using only standard apparatus and two fixed condensers with unknown residual parameters.

APPLICATIONS OF X-RAY TECHNIQUE TO INDUSTRIAL LABORATORY PROBLEMS.*

By H. P. Rooksby (Research Laboratories).
J. Roy. Soc. Arts, Vol. 88, 16th February, 1940.

This paper was presented before the Royal Society of Arts in January, 1940, and was awarded the Silver Medal. It was reprinted in the *G.E.C. Journal*, Vol. XI, No. 2, August 1940, p. 83.

The author discusses X-ray crystallography and gives many examples of its practical application in industry at the present time. Illustrations are given of the important fact that, in many cases, X-ray

* A limited number of reprints is available ; copies may be obtained on application to the Editor, G.E.C. Journal, Witton Engineering Works, B'ham, 6.

methods yield information unobtainable by chemical means, and that new conceptions of the ultimate structure of matter are arising directly out of recent measurements undertaken in the laboratory.

THE DETERMINATION OF ALUMINIUM, MAGNESIUM OR BERYLLIUM IN NICKEL ALLOYS.*
By R. C. Chirnside, L. A. Dauncey and P. M. C. Proffitt (Research Laboratories).
The Analyst, Vol. 65, p. 446, August, 1940.

A description is given of the results of an investigation into the most suitable methods of analysis for certain alloys used in the manufacture of radio valves and other thermionic devices.

THE DISTRIBUTION OF AUTELECTRONIC EMISSION FROM SINGLE CRYSTAL METAL POINTS. I. TUNGSTEN, MOLYBDENUM, NICKEL IN THE CLEAN STATE.*
By M. Benjamin and R. O. Jenkins (Research Laboratories).
Proc. Roy. Soc. A., Vol. 176, p. 262, October, 1940.

The field emission patterns from metal points show an electron distribution which can be related to the crystal lattice of the metal. A complete explanation is offered of the changes observed in the emission patterns in terms of the alterations of shape of the emitting point caused by the treatment to which it is subjected.

THE STABILITY OF REGENERATIVE CIRCUITS.*
By R. S. Rivlin (Research Laboratories).
Wireless Engineer, Vol. 17, p. 298, July, 1940.

Nyquist's theorem on the stability of regenerative circuits is discussed and an alternative proof is given.

THE EFFECT OF FLASH FREQUENCY ON THE APPARENT INTENSITY OF FLASHING LIGHTS HAVING CONSTANT FLASH DURATION.*
By A. E. Schuil (Research Laboratories).
Trans. I.E.S., Vol. 5, p. 117, September, 1940.

Observations were made on flashing lights having durations of 1/40th and 1/10th second. When the frequency is less than about 5 flashes per second, the flashes appear distinct and more intense than a steady light of the same total energy. Above about 13 flashes per second the light appears continuous but is no more economical than a steady light.

THE PHOTOMETRIC PROPERTIES OF SOME LUMINESCENT MATERIALS.*
By W. E. Harper, Margaret B. Robinson and J. N. Bowtell (Research Laboratories).
Trans. I.E.S., Vol. 5, p. 57, June, 1940.

Photometric data of some luminescent paints and lacquers are given which enable these materials to be treated as items of illuminating engineering equipment.

THE PHYSICAL REALISATION OF THE C.I.E. AVERAGE EYE.*
By G. T. Winch and C. F. Machin (Research Laboratories).
Trans. I.E.S., Vol. 5, p. 93, August, 1940.

There is described a multi-purpose apparatus which the authors have developed and which is suitable for use as an "average eye" as defined by the Comité International d'Eclairage. It can be adapted for use as a physical colorimeter and its possible general application to commercial photometry, colorimetry and colour rendering measurements is discussed.

THE EXAMINATION OF GLASS TANK-BLOCK REFRACTORIES BY X-RAY METHODS.*
By H. P. Rooksby and J. H. Partridge (Research Laboratories).
J. Soc. Glass Technology, Vol. 24, p. 109, June, 1940.

The technique of the X-ray powder method of analysis and its application to the study of glass tank-block refractories is discussed. One of the objects of the X-ray analysis is to find out if the burning treatment of tank-blocks has been sufficient to produce the structure required for present-day glass-melting tanks. The results of examinations of a large number of commercial blocks, both clay and "sillimanite," from various sources are included.

THE LIGHTING OF ROAD OBSTRUCTIONS.*
By H. H. Ballin and W. F. S. Debenham (Exterior Lighting Department).
Public Lighting, Vol. 5, No. 19, p. 75, October, 1940.

Under blackout conditions independent illumination of road obstructions such as islands and surface shelters has become imperative.

The Government regulations regarding the shape, position and intensity of suitable warning lights are discussed and suggestions made as to the requirements of a suitable fitting.

SHORT-WAVE VALVES.
By F. E. Henderson (Osram Valve Dept.).
Wireless World, February, 1940.

Certain effects that are quite innocuous on medium or long wavelengths are liable to give trouble when dealing with short waves. The article shows how these troubles may be minimised by the use of valves specially designed for short wave working. The article also deals with valves suitable for the range of 7·5 to 60 Mc/sec.; the relative importance of the effects arising from transit time of the electrons, inductance in the cathode lead, input impedance and frequency drift are briefly dealt with and comparative data are given for various valves to indicate these effects.

OVER-RUNNING MEDICAL LAMPS.
By C. T. A. Harper (Osram Lamp Works).
Electrical Review 126, p. 519, May 10th, 1940.

During surgical operations it is common practice to over-run small electrical inspection lamps, and it has been suggested that this might cause explosion since an ether-oxygen mixture may ignite at 100 degs. C. Curves of bulb temperature against applied voltage are given, from which it is clear that dangerous temperatures can easily be reached. A number of lamps were tested to destruction. In no case did the bulb burst, but this possibility cannot be ruled out.

PRINCIPLES AND APPLICATION OF ELECTRIC DISCHARGE LIGHTING.
By R. O. Ackerley (Illuminating Engineering Department).
Trans. Inst. Engineers-in-Charge, Vol. 46, Nos. 2 and 3, pp. 33-48 and 51-59.

The paper sketches the development of electric discharge lamps during the past ten years and summarises the types of lamps generally available, with qualities and characteristics from the practical lighting point of view. The economics of electric discharge lighting are discussed and the subject of quality and colour of light are considered. The author briefly discusses problems of installation of electric discharge lamps; the types of lamp covered include high pressure mercury plain and fluorescent lamps, sodium lamps, H.T. fluorescent tubes and mains burning fluorescent lamps.

* A limited number of reprints is available ; copies may be obtained on application to the Editor, G.E.C. Journal, Witton Engineering Works, B'ham, 6.

THE CALCULATION AND DESIGN OF ELECTRICAL APPARATUS. Second Edition.
By W. Wilson (Development Laboratory, Witton).
Chapman & Hall, 230 pp., 10/6.

This book has been considerably extended since its appearance five years ago. It is a treatise on the physical principles and mathematical formulae whereby the design of switchgear, control gear, electro-magnets, meters, instruments and similar apparatus, together with the busbars, connections, insulators and windings concerned can be calculated, and endeavours to do for static apparatus what numerous text books have done in the past for electrical machinery.

THE DESIGN AND TESTING OF HIGH POWER CIRCUIT BREAKERS.
By W. Wilson (Development Laboratory, Witton).
Electrical Engineer & Merchandiser (Australia), January and February, 1940, pp. 277—281 and 311—316.

A description is given of the various modern types of circuit breakers now available, and a critical study of the advantages and characteristics of each. This is followed by observations and calculations that require to be made in the course of high power testing, and calculations dealing with the arc, the behaviour of the oil, and with the various transient conditions.

SURGES AND SURGE GENERATORS.
By W. Wilson (Development Laboratory, Witton).
Electrical Engineer & Merchandiser, August, 1940, pp. 137—142.

The article first describes the nature and effects of lightning and other surges, and goes on to describe the methods of producing artificial lightning. The principle and the various types of surge generator are given, the article concluding with examples of their use in the testing of switchgear, lightning arresters and transformers.

FLUID FILLING-MEDIA FOR ELECTRICAL APPARATUS.
By F. Meyer (Development Laboratory, Witton).
Jour. I.E.E., Vol. 86, No. 520, p. 313, April, 1940.

This paper is reprinted on p. 210 of this issue of the *G.E.C. Journal*.

THE HIGH RUPTURING CAPACITY CARTRIDGE FUSE, WITH SPECIAL REFERENCE TO SHORT-CIRCUIT PERFORMANCE.
By J. W. Gibson (Development Laboratory, Witton).
(Reprinted in G.E.C. Journal, Vol. XI, Nos. 2 and 3, August, 1940 and February, 1941).

This paper was read before the Institution of Electrical Engineers in April, 1940. It is reprinted in the *G.E.C. Journal* in two parts; the first part appeared in Vol. XI., No. 2, August, 1940, and gives a summary of the advantages of the high-rupturing capacity cartridge fuse, and the breaking capacity requirements, as set forth in B.S.S. 88—1939, are enumerated. In the second part, which appears on p. 150 of this issue, conclusions are drawn as to the effect of circuit conditions on the duty imposed on the fuses; the choice of material is considered, and the article concludes with some notes on the application of these fuses.

REFRIGERATOR MOTORS.
By S. F. Philpott (Witton Engineering Works).
Electrical Review, July 26th, 1940.

Details the requirements of motors for domestic refrigerators, their construction and performance characteristics. Various manufacturers' products are described.

SHADED POLE MOTORS.
By S. F. Philpott (Witton Engineering Works).
Electrical Review, October 28th, 1940.

This article deals with the characteristics and construction of shaded pole motors; their advantages and limitations. Both standard and special types are described.

CIRCUIT BREAKING.
By C. J. O. Garrard (Witton Engineering Works).
Electrical Review, March 22nd, 1940.

The probable influence of advances in the theory of circuit breaking upon circuit breaker testing specifications is considered. It is concluded that fundamental alterations in the existing specifications are unlikely, although the production of economical breakers for very high voltages and the specification of tests for air blast breakers may demand more detailed knowledge of the manner in which the circuit constants influence the behaviour of the breaker.

The natural frequency, which is proportional to CL, is probably not the only parameter upon which the severity of the circuit depends. Another such parameter is the "stiffness" of the circuit, of which the ratio C/L is a measure. Two circuits may have the same natural frequency, but widely differing stiffness. In addition, the interaction of circuit and breaker is so close that the complete definition of circuit severity is impossible without regard to the characteristics of the breaker.

THE DESIGN OF LIQUID EARTHING RESISTANCES.
By C. J. O. Garrard (Witton Engineering Works).
Engineering, August 8th, 1940, p. 124.

A neutral earthing resistance must possess a high thermal storage capacity in order to absorb the requisite amount of energy in a small space and with moderate temperature rise. The liquid type of resistance introduced by the G.E.C. has many advantages over other types, such as cast iron grids or carbon plates. The basis of a satisfactory design is shown to be the reduction of the voltage gradient in the electrolyte to a figure of about 3,000 volts per inch and the prevention of boiling. Details are given of the electrical and mechanical design of resistances for service at voltages up to 66 kV.

FIREPROOF SWITCHGEAR.
By C. J. O. Garrard (Witton Engineering Works).
Electrical Industries, July, 1940.

A discussion of the relative security against fire provided by different forms of switchgear and of methods of reducing fire risk.